On the Theory of Radiation in Moving Bodies

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by Fritz Hasenöhrl.[1]

In the following, the attempt was undertaken to derive (from the thermodynamic laws) some theorems of the theory of heat radiation of moving bodies, when possible without the use of special assumptions concerning the essence of radiating heat, and then to apply them. By that, this work is essentially distinguished in terms of method and goal from some publications that appeared in recent times, which treat the same subject from the standpoint of the electromagnetic theory of light, especially on the basis of Lorentz's theory. However, the thermodynamic laws are not fully sufficient for the following. Namely, they don't provide the value for the radiation pressure upon a moving surface, which we will require in the following. The existence of such a pressure is a requirement of the second thermodynamic law, and its value for a stationary surface can also be derived from the energy theorem;[2] though this method fails when we try to apply it upon moving bodies. The value of radiation pressure upon a moving surface thus must be derived from a special hypothesis concerning the essence of radiating heat; namely we want to use the value derived by Abraham[3] from Lorentz's theory. Incidentally, the same value is also given, as we will see further below, from a quite different consideration based on the old theory of light.

We don't want to presuppose about the radiating heat anything other, than that it's a form of energy moving in all direction with the same absolute[4] velocity v, and that the direction of the energy flux, the "beam", is determined by the construction in accordance with Huyghens' principle, even when it is about the emission of a moving body.[5] Furthermore we naturally presuppose the unrestricted validity of the thermodynamic laws.

Despite of the essentially different starting point, still some points of contact with the electromagnetic theory will be given, especially with the work of Abraham[6] which appeared when the present work was already finished in the main. This mostly concerns various purely geometrical considerations, although they are not always so simple of not being capable to give rise to misunderstandings. As far as I can see, there is complete agreement between Abraham and me.

The content of the present work is shortly as follows: In § 1, the geometric relations between absolute and relative beam velocity and direction are stated. They partially have been stated already by other authors (as mentioned), though they had to be completely compiled here.

§ 2 gives the definitions for the concepts "absolute radiation", "total" and "true relative radiation"; also these concepts were (partly with another meaning) already formed by Abraham.[7] Furthermore the theorem is derived, that the "true relative" radiation of a moving black body obeys Lambert's cosine theorem.

In § 3, the density of radiation in a moving cavity is calculated. It consists of two parts, of which one is stemming from the heat reservoir of the boundary of the cavity, while the other one is the equivalent of the work required to set such a system into motion.

Up to now, only the existence of radiation pressure was presupposed. Since we also have to know the amount of it, we have to take (in § 4) the value of the radiation pressure from the works of Abraham, and also another derivation of the value (as mentioned before) is given.

In § 5 the work is calculated, which must be performed to change the velocity of a cavity filled with radiation. The work that must be performed to increase the velocity, is gained again when it is decreased by the same quantity, in case the velocity change is carried out infinitively slow. Then this process is thus reversible: namely both at "isothermic" as well as at "adiabatic" change of velocity. These relations give rise to the formation of the concept of an apparent mass caused by radiation, which is quite analogous to the so-called electromagnetic mass.

In § 6 we concern ourselves with the quantity of heat, given off by a moving black body to a cavity rigidly connected with it, when the velocity of this system is changed. There, we come to a contradiction with the second law of thermodynamics, which can be solved by assuming, that the dimensions of matter are depending on the velocity of their translatory motion through the aether. Namely, exactly the amount for this dependency is given, which according to Lorentz and Fitzgerald is necessary for the explanation of the negative result of the experiment of Michelson and Morley.

§ 1.

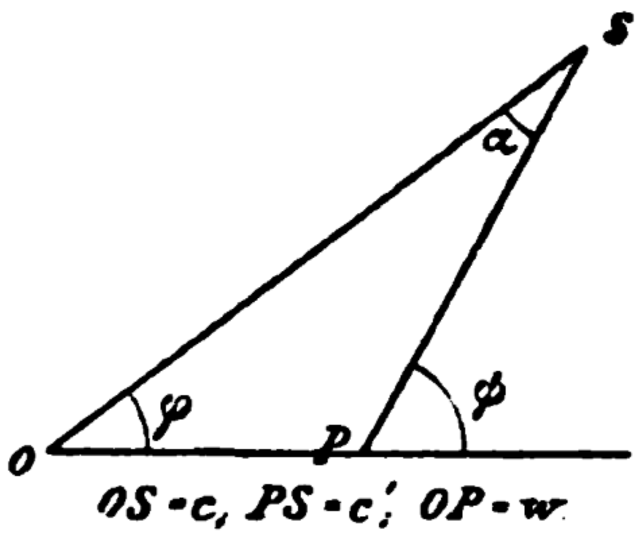

Fig. 1.

It's known that it is convenient in the optics of moving bodies to use two reference systems, of which one is at absolute rest while the other one is in relative rest.[8] Accordingly one has to distinguish between absolute and relative velocity and direction of a beam. Let the velocity of matter be given by the vector $\mathfrak{w}$, whose amount be $w = \beta c$. (c is the speed of light.) Furthermore, we denote by φ the *acute* angle enclosed by a line (coinciding with the *absolute* beam direction) with the vector $\mathfrak{w}$; then by ψ the *acute* angle (enclosed by a line coinciding with the *relative* beam direction) with $\mathfrak{w}$. Furthermore, let c' be the amount of the relative velocity of light, then from Fig. 1 the following relations are easily given, in which the above or below sign is valid, depending on whether the direction of motion of matter and that of the considered beam have the same sense or not:

(1)
$$c' = c\sqrt{1+\beta^2 \mp 2\beta\cos\varphi}$$

(2)
$$= c\left(\mp\beta\cos\psi + \sqrt{1-\beta^2\sin^2\psi}\right).$$

We also will often use the notation:

(3a)
$$c_- = c\sqrt{1+\beta^2 - 2\beta\cos\varphi} = c\left(-\beta\cos\psi + \sqrt{1-\beta^2\sin^2\psi}\right)$$

(3b)
$$c_+ = c\sqrt{1+\beta^2 + 2\beta\cos\varphi} = c\left(\beta\cos\psi + \sqrt{1-\beta^2\sin^2\psi}\right)$$

Furthermore it is easily given:

(4)
$$\sin\psi = \sin\varphi\frac{c}{c'},$$

(5)
$$\cos\psi = (\cos\varphi \mp \beta)\frac{c}{c'},$$

(6)
$$\cos\varphi = \pm\beta\sin^2\psi + \cos\psi\sqrt{1-\beta^2\sin^2\psi},$$

$$1 \mp \beta \cos \varphi = \sqrt{1 - \beta^2 \sin^2 \psi} \frac{c'}{c}. \tag{7}$$

We denote the aberration angle (the angle between the absolute and relative beam direction) by α ; it is

$$\sin \alpha = \beta \sin \psi, \ \cos \alpha = \sqrt{1 - \beta^2 \sin^2 \psi}. \tag{8}$$

Finally, it is given by differentiation of equation (6):

$$\sin \varphi \ d\varphi = \sin \psi \ d\psi \frac{c'^2}{c^2 \cos \alpha}, \tag{9}$$

an equation that povides the relation of the opening angle of a light pencil in terms of absolute and relative beam path.

If the mirror is moving with arbitrary velocity in the direction of its normal (or also perpendicular to it), then the law of reflection for the relative beam direction is strictly satisfied.[9]

Under "relative" we will always understand "relative to a system moving moving with velocity $\mathfrak{w}$. When we are dealing with different velocities of the reference system, we will say in short: relative "with respect to $\mathfrak{w}$ ".

§ 2.

As the positive sense of the normal, or simply as the normal of the surface element of a material body, we want – once and for all – define the direction, which is directed from the body to the outside (into the surrounding aether). By that, also the positive sense of the normal at a radiating or reflecting surface element is given. Now, if the surface element is moving in the sense of the positive or negative normal, then in the following we want to speak in short about its positive or negative motion.

Under *absolute radiation* one understands the energy quantity, which traverses (in unit time) the unit surface of an absolutely resting plane situated perpendicular of the absolute beam direction. This energy quantity is equal to the heat, absorbed by the unit surface of an equally located black plane.

Under *total relative radiation* we understand the energy quantity, which traverses (in unit time) the unit surface of an (imagined) plane, moving with absolute velocity $\mathfrak{w}$, and which is oriented perpendicular to the relative beam direction. If this imagined plane is replaced by a material black plane (moving and oriented in the same way), then this total relative radiation is not identical with the amount of the heat absorbed by the latter; because the work of the radiation pressure or of the (external) work against it, has to be added here additionally, depending on whether the black plane is moving in the negative or positive sense. If one decreases or increases the total relative radiation by the amount of this work, then one obtains the amount of heat absorbed by the black plane, which amount we want to call the *true relative radiation*.

This mode of conception, according to which mechanical work is directly transformed into radiation energy and *vice versa*, was first spoken out (as I believe) by Poynting[10] and then by v. Türin,[11] independent from any specific idea concerning the essence of radiating heat, and purely as the consequence of the energy theorem. This is also in agreement with the theory of Abraham. What we call here true relative radiation, simply corresponds to the "relative radiation" of Abraham.

Quite analogous are the relations at the emission of a moving black surface. It was left by a certain amount of total relative radiation, which is equal to the true relative radiation and is increased or decreased by the amount of work performed by or against the radiation pressure, depending on whether the black surface is moving in the positive or negative sense. Only the true relative radiation stems from the heat reservoir of the moving body, by which our terminology may be justified.

We want to denote the difference between the total and true relative radiation, as the *apparent relative radiation*.

The relation between absolute and total relative radiation is given from purely geometric considerations. Since the density of the radiation energy is a scalar quantity, it has naturally the same value for both reference systems. Thus if we are dealing with parallel radiation, then the relation of absolute radiation to the total relative one is $c : c'$.

If we now consider a light pencil, whose absolute beam direction enclose angles with $\mathfrak{w}$, whose magnitude lies between φ and $\varphi + d\varphi$, and if the absolute radiation of this pencil is for example given by $2\pi\ J\ sin\varphi\ d\varphi$, then the corresponding total relative radiation is $2\pi\ J\ sin\varphi\ d\varphi\ c'/c$ according to the above; if we write this in the form $2\pi\ J'\ sin\psi\ d\psi$, then the corresponding equation (9) is:

(10)
$$J' = J \frac{c'^3}{c^3 \cos\alpha}$$

Thus if we consider an arbitrary scattered radiation, then the "radiation intensities" of the absolute and total relative radiation behave as $1 : c'^3/c^3\ \cos\alpha$; the densities of radiation, moving in the absolute and relative beam path in the unit of the opening angle, behave as

(11)
$$\frac{J}{c} : \frac{J'}{c'} = 1 : \frac{c'^2}{c^2 \cos\alpha}.$$

We now want to find out the relation between the total and true relative radiation. For this purpose, we consider a cylindric cavity R, whose base surfaces A and B shall belong to two black bodies, while the shell surface of the cylinder, as well as the external boundary of both black bodies, shall be perfectly reflecting surfaces. The cross-section of space R shall be equal to 1; its height equal to h. Both space R as well as the exterior space shall be totally free of ponderable matter. This system shall move with velocity w in the direction of the arrow (Fig. 2). The exterior space shall have the temperature 0° A (so that the system experiences no external resistance in its motion), while the black surfaces A and B shall have a certain temperature different from zero. (When the system described is at rest, it is completely closed with respect to the outside.)

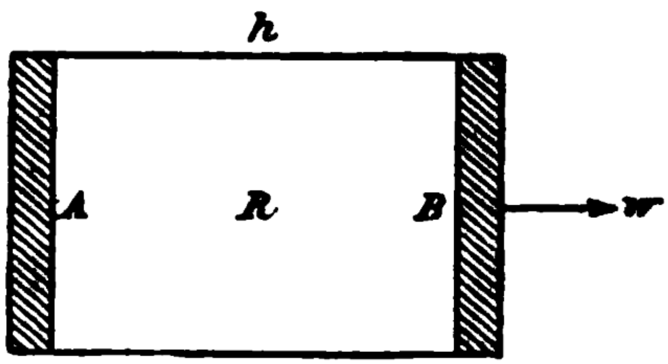

Fig. 2.

Now, surface A is left in unit time by a certain amount of total relative radiation; we pick out the radiation whose relative direction encloses angles between ψ and $\psi + d\psi$ with the normal (and thus also with $\mathfrak{w}$)). Let its amount be:

(12a)
$$2\pi\ i\ \cos\psi\ \sin\psi\ d\psi.$$

This radiation exerts a certain pressure upon A, whose component coinciding with the direction of the negative normal, we want to denote by:

(13)
$$2\pi\ p_1\ \cos\psi\ \sin\psi\ d\psi.$$

Since A is moving in the positive sense, the work

(14)
$$w \cdot\ 2\pi\ p_1 \cos\psi\ \sin\psi\ d\psi$$

must be performed form the outside in unit time, so that the uniform motion of surface A remains conserved. If we denote the true radiation of A by

(15) $$2\pi\ i_0\ \cos\psi\ \sin\psi\ d\psi.$$

then according to the things said earlier:

$$i = i_0 + wp_1.$$

The emphasized total relative radiation (12a) now hits plane B moving in the negative sense, and there it will be partly transformed in mechanical work and partly it is absorbed. Namely, the relation of these two parts depends on the magnitude of the pressure exerted upon this surface by the radiation (12a) incident in B. So if we think for a moment about B as having temperature O° A., then the radiation emanating from A is the only one present in R, and the reaction principle requires (whose denial in this case would be contradicting all views expressed up to now) that radiation (12a) exerts the same pressure in B as in A, though in opposite direction.[12] Since the corresponding work, whose amount is again given by (14), is performed by the radiation here, then its amount is to be subtracted from (12a) to obtain the radiation absorbed in B, for which we again obtain the true radiation (15).

If we assume again, that B has a certain temperature different from zero, then under a certain angle it is left by the total relative radiation

(12b) $$2\pi\ i\ \cos\psi\ \sin\psi\ \ d\psi\,,$$

to which the quantity of work

$$w \cdot 2\pi\ p_2\ \cos\psi\ \sin\psi\ \ d\psi$$

is to be added in this case, to obtain the true radiation provided by B; as before, it provides the quantity of heat absorbed by A at the other side of the cavity.

Then, the same though opposite pressure, is present at both sides:

$$2\pi\ (p_1 + p_2)\ \cos\psi\ \sin\psi\ \ d\psi$$

We see, that the true relative radiation is solely decisive for the heat transport between A and B; because it completely stems from the heat reservoir of one black body, and is completely transformed into the heat reservoir of the other one. The other energy present in R, the *apparent radiation*, is gained from mechanical work. At one side of the cavity, heat is steadily transformed into work, which traverses it and is retransformed at the other side as work of the same amount, so that altogether no work is performed or gained at uniform translation of our system.

If both surfaces A and B are of the same temperature, then the second thermodynamic law demands that the total, mutual, true, and relative radiation shall be the same; because only then the temperatures remain mutually the same. In order, that the same is also valid for two arbitrary surface elements which are arbitrarily oriented, the cosine law must hold for the true relative radiation; *i.e*, i_0 must be independent of ψ, and furthermore it must have the same value for A and B, thus for a positively and negatively moving surface. Thus we obtain the theorem (which is important for the following):

The true relative radiation of a moving black body, obeys (in the relative beam path) Lambert's cosine law.

On the other hand, the total relative radiation intensity of a black surface moving in the positive or negative sense, has the amount

(17a) $$i = i_0 + wp_1$$

or

(17b) $$i' = i_0 + wp_2,$$

namely there is no reason to assume that these magnitudes are independent from ψ as well.

It can be of no influence in R,[13] when we replace one of the two black surfaces A or B by a mirror (or also by any other surface). Especially, also (16) gives the value for the pressure upon a moving mirror, when (moving in the negative or positive sense) it is hit by the total relative radiation (12a) or (12b).

§ 3.

We now want to calculate the energy in our moving cavity. The radiation (12a) emanated from A is moving with velocity c_- according to § 1, and radiation (12b) is moving with velocity c_+ through the cavity. Since the path to be traveled by both rays in cavity R is $h/\cos\psi$, then the time in which they are in R, is equal to $h/c_-\cos\psi$ or $h/c_+\cos\psi$; they thus contribute the summand

$$h\cdot 2\pi\sin\psi\ d\psi\left(\frac{i}{c_-}+\frac{i'}{c_+}\right)$$

to the energy content of R. When we integrate this expression with respect to ψ from 0 to $\pi/2$, furthermore dividing by the volume h of space R, then we obtain the density ϱ of the total radiation in R. If we insert for i and i' their values from (17a) and (17b), then it becomes:

$$\varrho=2\pi\int_0^{\pi/2}\sin\psi\ d\psi\left(\frac{i_0+wp_1}{c_-}+\frac{i_0-wp_2}{c_+}\right).$$

If we now put

(18) $$\varrho=\epsilon+\epsilon',$$

where

$$\epsilon=2\pi i_0\int_0^{\pi/2}\sin\psi\ d\psi\left(\frac{1}{c_-}+\frac{1}{c_+}\right)$$

and

(19) $$\epsilon'=2\pi w\int_0^{\pi/2}\sin\psi\ d\psi\left(\frac{p_1}{c_-}-\frac{p_2}{c_+}\right)$$

If we insert the values from (3a) and (3b) for c_- and c_+ in the first of these expression, it becomes

$$\epsilon=\frac{4\pi i_0}{c(1-\beta^2)}\int_0^{\pi/2}\sin\psi\ d\psi\sqrt{1-\beta^2\sin^2\psi}.$$

We now set the density of energy in a resting cavity

(20) $$\frac{4\pi i_0}{c}=\epsilon_0\left(=\frac{4e}{c}\right)$$

(e is the "emission capacity") and

$$\varkappa=\frac{1}{1-\beta^2}\int_0^{\pi/2}\sin\psi\ d\psi\sqrt{1-\beta^2\sin^2\psi}$$

or after execution of the integration

(21) $$\varkappa=\frac{1}{2(1-\beta^2)}+\frac{1}{4\beta}\log\frac{1+\beta}{1-\beta}.$$

Then it is

(22) $$\epsilon = \epsilon_0 \varkappa$$

When we neglect magnitudes beginning with order β^4, it becomes

(23) $$\varkappa = 1 + \frac{2}{3}\beta^2.$$

Thus, ϵ is evidently the density of true radiation. In order to calculate ϵ', *i.e.*, the density of the apparent radiation, we have to know the values of p_1 and p_2, with which we want to concern ourselves in the following section.

§ 4.

According to Abraham, the radiation pressure upon a moving plane is equal to the radiation incident or emanating in unit time (in our terminology, this is the total relative radiation) divided by the speed of light; namely, this pressure is acting in the direction of the absolute propagation in the sense of the negative normal. Since we understood p_1 and p_2 as perpendicular pressure components, it is thus:

$$2\pi p_1 \cos\psi \sin\psi \; d\psi = \frac{2\pi i \cos\psi \sin\psi \; d\psi}{c} \cdot \cos\varphi,$$
$$2\pi p_2 \cos\psi \sin\psi \; d\psi = \frac{2\pi i' \cos\psi \sin\psi \; d\psi}{c} \cdot \cos\varphi.$$

If we divide away the same factors at both sides, and if we insert for $\cos\varphi$ its value from (6), then under consideration of the corresponding sign:

(24a) $$p_1 = \frac{i}{c}\left(\beta \sin^2\psi + \cos\psi\sqrt{1-\beta^2\sin^2\psi}\right),$$

(24b) $$p_2 = \frac{i'}{c}\left(-\beta \sin^2\psi + \cos\psi\sqrt{1-\beta^2\sin^2\psi}\right).$$

If we insert this into equations (17), then we obtain

(25a) $$i = \frac{i_0}{\sqrt{1-\beta^2\sin^2\psi}\left(-\beta\cos\psi + \sqrt{1-\beta^2\sin^2\psi}\right)} = \frac{i_0 c}{\cos\alpha \cdot c_-},$$

(26a) $$i' = \frac{i_0}{\sqrt{1-\beta^2\sin^2\psi}\left(\beta\cos\psi + \sqrt{1-\beta^2\sin^2\psi}\right)} = \frac{i_0 c}{\cos\alpha \cdot c_+}.$$

Thus we can set:

(26a) $$\begin{cases} p_1 = \dfrac{i_0\left(\beta\sin^2\psi + \cos\psi\sqrt{1-\beta^2\sin^2\psi}\right)}{c\sqrt{1-\beta^2\sin^2\psi}\left(-\beta\cos\psi+\sqrt{1-\beta^2\sin^2\psi}\right)} \\ \quad = \dfrac{i_0\left(\cos\psi + \beta\sqrt{1-\beta^2\sin^2\psi}\right)}{c(1-\beta^2)\sqrt{1-\beta^2\sin^2\psi}}, \end{cases}$$

(26b)
$$\begin{cases} p_2 = \dfrac{i_0\left(-\beta\sin^2\psi + \cos\psi\sqrt{1-\beta^2\sin^2\psi}\right)}{c\sqrt{1-\beta^2\sin^2\psi}\left(\beta\cos\psi + \sqrt{1-\beta^2\sin^2\psi}\right)} \\ \quad = \dfrac{i_0\left(\cos\psi - \beta\sqrt{1-\beta^2\sin^2\psi}\right)}{c(1-\beta^2)\sqrt{1-\beta^2\sin^2\psi}}. \end{cases}$$

These expressions can now (as already mentioned in the introduction) also be derived from another hypothesis; namely from the assumption, that a moving body emanates in unit time the same amount of waves as at rest, and that also the amplitude of these waves is not changed by the motion of the light source. Only the wavelength experiences a change in accordance with Doppler's principle. If one now assumes in accordance with the old theory of light, that the energy of a wave train is *caeteris paribus* inversely proportional to the square of the wavelength per unit length, *i.e.*, that one energy quantity (which is inversely proportional to the first power of its length) is connected to one wave, then (since the number of emanated waves is the same in both cases) the radiation of the moving body is related to that of the resting one, by the inverse proportion of the wavelengths. Since this proportion is equal to $(1 \mp \beta\cos\varphi)$ according to Doppler's principle, then we obtain:

$$i = \frac{i_0}{1-\beta\cos\varphi} = \frac{i_0 c}{\cos\alpha \cdot c_-}$$

or

$$i' = \frac{i_0}{1+\beta\cos\varphi} = \frac{i_0 c}{\cos\alpha \cdot c_+},$$

where we have used (7). These expressions are now indeed identical with equations (25). If we insert them into equations (17), we of course can also derive equations (24) and (26).

The thought process stated here, was first applied by Larmor[14] to the problem of reflection, and then by Poynting[15] to the emission of a moving surface. The mentioned authors confine themselves, however, to the case of perpendicular incidence or emission. Incidentally, Poynting[15] has calculated by a different method the radiation pressure at perpendicular emission with the same result.

If we insert values (26) into (16), then we obtain the pressure upon the surfaces A and B. For example, if we calculate the pressure acting upon surface B, and express this value for example by the density of the total incident radiation

$$\frac{2\pi\ i\ \sin\psi\ d\psi}{c_-} = d,$$

then we obtain

(27)
$$2d\cdot\frac{c_-^2}{c^2}\frac{\cos^2\psi}{1-\beta^2} = 2d\frac{(\cos\varphi-\beta)^2}{1-\beta^2},$$

which of course is in agreement with the corresponding one of Abraham.[16]

The pressure upon a moving mirror incidentally can be derived very easily from the mentioned hypothesis, and one also obtains the law of reflection with one stroke.[17] It follows indeed immediately from it, that the intensities of the incident and reflected light behaves inversely as the wavelengths, and directly as the oscillation numbers.[18]

§ 5.

Now we want to concern ourselves with the fraction of the energy in R, which is due to the apparent radiation and which is gained from mechanical work. If we set into (19) the values from (26) for p_1 and p_2, it becomes

$$\epsilon' = \frac{2\pi w i_0}{c^2(1-\beta^2)^2}\int_0^{\pi/2} \sin\psi\ d\psi \frac{\beta\cos^2\psi + \beta(1-\beta^2\sin^2\psi)}{\sqrt{1-\beta^2\sin^2\psi}}.$$

We put

(28) $$\epsilon' = \epsilon_0 \tau,$$

where ϵ_0 has the meaning given by (20), while we obtain for τ after execution of the integration:

(29) $$\tau = \frac{1+\beta^2}{2(1-\beta^2)^2} - \frac{1}{4\beta}\log\frac{1+\beta}{1-\beta}.$$ [19]

When we neglect terms starting with order β^4, then it becomes

(30) $$\tau = \frac{4}{3}\beta^2.$$

As already mentioned several times, the energy quantity $h\epsilon'$ present in space R, is gained from mechanical work. Yet, since no work is altogether performed at uniform translation of our system, then this energy quantity must be the equivalent of a work which is performed at the acceleration of it, and which is thus added to the work against the ordinary resistance of inertia. One can easily illustrate this to oneself in the following way: Imagine a system being at rest in the beginning, and the surfaces are somehow hindered of radiating energy, so that space R contains no energy. Now, if we bring the system suddenly to velocity w, and simultaneously set free the radiation of A and B, then the work $w \cdot 2\pi\ p_1\ \cos\psi\ \sin\psi\ d\psi$ (14) in unit time must immediately be performed against the radiation emanating from A. As soon as the radiation arrives in B, the same amount of work is gained there; though the time $h/c_- \cos\psi$ passes until this happens; during this time, the external work (15) is thus uncompensated. Accordingly, the radiation emanating from B immediately performs the work $w \cdot 2\pi\ p_2\ \cos\psi\ \sin\psi\ d\psi$, and now the time $h/c_+ \cos\psi$ passes before this work is compensated by an equal amount of work of the external forces in A. Thus altogether, the work

$$2\pi\cos\psi\sin\psi\ d\psi\ w\left(p_1\frac{h}{c_-\ \cos\psi} - p_2\frac{h}{c_+\ \cos\psi}\right)$$

must be provided from the outside. If we integrate this expression from 0 to $\pi/2$ and if we consider (19), then we recognize that the performed work has indeed the amount $h\epsilon'$.

Now, when our system was initially at rest, yet the surfaces A and B were not hindered to emanate, then the energy density ϵ_0 is initially present in R, and it is the question, what is happening with the energy quantity when the system is suddenly brought to velocity w. The previous consideration indicates that the energy amount has to be completely absorbed by the surfaces A and B; no part of it may be transformed into mechanical work. Incidentally, also a direct calculation leads to this result, which starts from the values (given in the previous section) for the quantities p_1 and p_2. This calculation is only a special case of the following one, and therefore I don't want to execute it here, since this already happened at another place.[20]

On the other hand, if we would bring our system (with the energy given above in the cavity) suddenly from the state of motion into that of rest, then the whole energy quantity $h\epsilon'$ (generally the whole surplus of energy in R caused by motion) would be absorbed by A and B; because at the now resting boundary surfaces of space R, no work can indeed be performed. Thus if the motion of our system is suddenly accelerated, then work must be performed which is transformed into (radiating) heat; this work won't be regained at a sudden delay of motion, its energy amount rather remains in the form of heat. Such rapid changes of the velocity of motion are thus accompanied with irreversible processes. On the other hand, the energy transformation is completely reversibly, as I will show soon. The relations are quite similar, for example, as at the compression and expansion of a gas. The compression work is always transformed into heat, independently on whether the compression takes place slowly or rapidly; yet this heat quantity only then is transformed into work again, when the expansion takes place quite slowly, *i.e.*, "reversible".

Thus we want to investigate now, what is happening when we change the velocity w of our system by an infinitely small value δw. (It can of course be presupposed, that these infinitely small changes occur suddenly.)

Let

$$w + \delta w = w_1 = \beta_1 c = (\beta + \delta\beta)c.$$

Thus at the beginning, within R there is a total relative radiation (with respect to w) of radiation intensity i or i'. According to (10), this radiation is corresponding to an absolute radiation of intensity

$$i \cdot \frac{c^3 \cos\alpha}{c_-^3} \text{ or } i' \cdot \frac{c^3 \cos\alpha}{c_+^3}.$$

And to this radiation, a total relative radiation with respect to $w + \delta w$ is corresponding again, of intensity

$$i \cdot \frac{c^3 \cos\alpha}{c_-^3} \cdot \frac{c_{-,1}^3}{c^3 \cos\alpha_1} \text{ or } i' \cdot \frac{c^3 \cos\alpha}{c_+^3} \cdot \frac{c_{+,1}^3}{c^3 \cos\alpha_1},$$

where the index 1 supplemented to the quantities c_-, c_+, α means, that these quantities are to be formed by ψ_1 and β_1 instead of ψ and β.

Thus we can say: At the beginning, the total relative radiation in R with respect to w_1, is given by the expressions

$$2\pi \cos\psi_1 \sin\psi_1 d\psi_1 i \frac{c_{-,1}^3 \cos\alpha}{c_-^3 \cos\alpha_1}$$

or

$$2\pi \cos\psi_1 \sin\psi_1 d\psi_1 i' \frac{c_{+,1}^3 \cos\alpha}{c_+^3 \cos\alpha_1}.$$

The density of these radiations is obtained by division by $c_{-,1}\cos\psi_1$ or $c_{+,1}\cos\psi_1$. Thus the energy amount of these emphasized radiations in R is equal to (density times volume):

(31a)
$$2\pi \sin\psi_1 d\psi_1 i \frac{c_{-,1}^2 \cos\alpha}{c_-^3 \cos\alpha_1} \cdot h$$

or

(31b)
$$2\pi \sin\psi_1 d\psi_1 i' \frac{c_{+,1}^2 \cos\alpha}{c_+^3 \cos\alpha_1} \cdot h.$$

Now, the relation of the total to the true relative radiation (the latter is actually absorbed) was equal to i/i_0 or i'/i_0; thus when the velocity is equal to $c/c_- \cos\alpha$ or $c/c_+ \cos\alpha$. When the velocity has now the value w_1, then this relation becomes equal to $c/c_{-,1}\cos\alpha_1$ or $c/c_{+,1}\cos\alpha_1$. Thus, from the radiation (31) present in R at the beginning, the fraction

$$2\pi \sin\psi_1 d\psi_1 i h \frac{c_{-,1}^3 \cos\alpha}{c \cdot c_-^3}$$

or

$$2\pi \sin\psi_1 d\psi_1 i' h \frac{c_{+,1}^3 \cos\alpha}{c \cdot c_+^3}$$

is now absorbed. If we want to obtain the fraction Q of the whole energy contained in R at the beginning, which is absorbed by A and B after a velocity change of δw, then we have to integrate these expressions with respect to ψ_1 from 0 to $\pi/2$. We preliminarily insert the values from (25) for i and i', and thus we obtain

$$Q = \int_0^{\pi/2} 2\pi \sin\psi_1 d\psi_1 i_0 h \left(\frac{c_{-,1}^3}{c_-^4} + \frac{c_{+,1}^3}{c_+^4}\right).$$

Now it is according to (1):

$$c_{-,1}^2 = c^2 + (w + \delta w)^2 - 2c(w + \delta w)\cos\varphi = c_-^2 - 2\delta w(c\ \cos\varphi - w),$$

or by use of (5)

$$c_{-,1}^2 = c_-^2 - 2\delta w c_- \cos\psi.$$

Since we can also set $c_{-,1}\cos\psi_1$ instead of $c_-\ \cos\psi$ (within the expression multiplied with the infinitely small magnitude δw) it eventually becomes:

$$c_- = c_{-,1}\left(1 + \delta w \frac{\cos\psi_1}{c_{-,1}}\right).$$

Quite analogously it is given:

$$c_+ = c_{+,1}\left(1 - \delta w \frac{\cos\psi_1}{c_{+,1}}\right).$$

If we use these equations, it becomes.

$$Q = 2\pi i_0 h \int_0^{\pi/2} \sin\psi_1 d\psi_1 \left(\frac{1 - 4\delta w\frac{\cos\psi_1}{c_{-,1}}}{c_{-,1}} + \frac{1 + 4\delta w\frac{\cos\psi_1}{c_{+,1}}}{c_{+,1}}\right).$$

or

$$Q = 2\pi i_0 h \int_0^{\pi/2} \sin\psi_1 d\psi_1 \left(\frac{1}{c_{-,1}} + \frac{1}{c_{+,1}}\right)$$
$$-8\pi i_0 h \delta w \int_0^{\pi/2} \sin\psi_1 \cos\psi_1 d\psi_1 \left(\frac{1}{c_{-,1}^2} - \frac{1}{c_{+,1}^2}\right).$$

We can determine the first integral by the aid of equation (21), the second one has the value:

$$\frac{2\beta_1}{c^2(1-\beta_1^2)^2}\int_0^{\pi/2} \sin\psi_1 \cos^2\psi_1 d\psi_1 \sqrt{1 - \beta_1^2 \sin^2\psi_1}$$
$$= \frac{2}{c^2(1-\beta_1^2)^2}\left(\frac{1+\beta_1^2}{8\beta_1} - \frac{(1-\beta_1^2)^2}{16\beta_1^2}\log\frac{1+\beta_1}{1-\beta_1}\right),$$

thus it becomes

$$Q = h\epsilon_0 \varkappa_1 - h\epsilon_0 \frac{\delta w}{c}\left(\frac{1+\beta_1^2}{2\beta_1(1-\beta_1^2)^2} - \frac{1}{4\beta_1^2}\log\frac{1+\beta_1}{1-\beta_1}\right),$$

where $\varkappa_1$ is the value, which emerges from $\varkappa$ when β_1 is replaced by β (see equation (21)). One easily convince oneself, that the bracket expression in the last equation has the value $d\varkappa_1/d\beta_1$; thus it becomes:

$$Q = h\epsilon_0\left(\varkappa_1 - \delta\beta\frac{dx_1}{d\beta_1}\right) = h\epsilon_0\varkappa = h\epsilon.$$

Our earlier assertion is proven by that. At the beginning, when the velocity of our system was w, the energy quantity contained in B had the amount $h(\epsilon + \epsilon') = h\epsilon_0(\varkappa + \tau)$; when the velocity is changed, and the equilibrium of radiation in R is reestablished, then this energy quantity has the amount $h\epsilon_0(\varkappa_1 + \tau_1)$(where τ_1 is formed from β_1 in the same way again, as τ from β). As we know, the fraction $h\epsilon_0\varkappa_1$ of the heat reservoir of bodies A and B stems from that; since they (as we have just proven) absorb the fraction $h\epsilon_0\varkappa$ from the initially existing radiation, we see that when the velocity is changed from w to w_1, the boundary of the cavity gives off the heat

$$h\epsilon_0(\varkappa_1 - \varkappa)$$

to the latter, while the difference of the amount of apparent radiation, thus the energy quantity

$$h\epsilon_0(\tau_1 - \tau),$$

is the equivalent of the work to be performed in order to accelerate our system. Namely, the sign of δw played no role at all in our reasoning; the process is thus reversible. The same is also true for a finite velocity change, which one indeed can imagine as caused by several repetitions of arbitrarily small changes of that kind. It's only necessary that it must be executed so slowly, that the radiation in R is always in the state of equilibrium corresponding to the momentary value of velocity. Only then, the energy transformations accompanying the finite velocity change, are reversible.

This work, which is performed at any velocity increase, and gained at any velocity decrease, and which of course is added to the work against the inertial resistance in the ordinary sense, shows us that the ordinary kinetic energy of our system appears to be increased by the amount $h\epsilon_0\tau$; the state of facts is such, as if the mass of our system were increased by the amount $2h\epsilon_0\tau/w^2$. If we insert herein for τ its approximate value form (30), then this apparent increase of mass becomes independent from velocity, and namely equal to:

(32) $$\frac{8}{3}\frac{h\epsilon_0}{c^2}.$$

Namely, the introduction of this concept of an apparent mass caused by radiation, is completely analogous to the introduction of the electromagnetic mass. We also mention, that the relation of this apparent mass to the energy of the resting cavity is equal to $8/3c^2$: the relation of the electromagnetic mass of a spherical electron to the energy of the resting electron is (in the case of surface charge) equal to $4/3c^2$,[21] thus of the same order of magnitude. Incidentally, also the exact values of the apparent mass (see equation (29)) show in both cases similarities in their form; however, they are not identical.

It is to be mentioned that we have tacitly assumed, that the quantity i_1 and thus ϵ_0 is not (explicitly) dependent from the value of the translatory velocity; we will come back later to this.

For providing the values of quantity τ and thus the apparent mass just introduced, it was necessary to know the quantities p_1 and p_2. Yet also without it, one can conclude the existence of them from expression (19); furthermore one immediately recognizes, that this expression must be proportional to w^2 in first approximation.

Since the heat content of every body partly consists of radiating heat, the things that we have demonstrated at a cavity, are true *mutatis mutandis* for every body whose temperature is different from 0° A.. In particular, every body must have an apparent mass determined by the inner radiation, and which is therefore above all dependent on the temperature.

The velocity changes just considered, and thus the state of radiation in our cavity, can be denoted as isothermic ones, since space R was always in connection with the bodies A and B, whose heat capacity was assumed by us as infinitely great against that of space R itself.

If we want to study the process at "adiabatic" changes, then it would be the most simple way, to consider a space filled by radiation energy of certain amount and which is enclosed at all sides by mirrors, and then giving to it different velocities. Though the execution of this thought gives the result, that radiation states are formed in this case, which are essentially different from the ones considered earlier, and which we can denote as unstable; because at the slightest change of the surface constitution of the boundary surfaces, the radiation state immediately becomes different, while the states considered earlier were quite independent from the constitution of the boundary of the cavity.

To avoid this difficulty, we want to imagine adiabatic changes of the radiation state in our cavity as of such kind, that a black body is present in the space surrounded at all sides by mirrors, yet whose capacity is so small, that its heat content can be neglected against the energy content of space R. The purpose of it is only to regulate the distribution of radiation in different directions, so that they remain in stable equilibrium when the velocity of the system is changed; simultaneously, it can serve to define the temperature of the radiation on the moving cavity, by setting the latter equal to the temperature of this small body.

Now we consider such a space moving with velocity w, which for example was earlier in connection with an extended black body, which had the same velocity (thus with a heat reservoir of certain temperature). Then the energy density

$$\epsilon + \epsilon' = \epsilon_0 \varkappa + \epsilon_0 \tau.$$

is present in this space. Now, if the velocity is changed by δw to $w\epsilon_1$, then the black body absorbs the fraction $h\epsilon = h\epsilon_0\varkappa$ from the energy $h(\epsilon + \epsilon')$ initially present. (Let h again be the volume of that space). However, due to the vanishing capacity of that body, this heat quantity must be equal to the heat given off by it when the velocity is changed. Yet according to the things said earlier, the latter must be proportional to $\varkappa_1$[22]; thus ϵ_0 must have been changed, for example to $\epsilon_{0,1}$, so that

$$\epsilon_{0,1} = \epsilon_0 \varkappa .$$

The amount of true radiation thus remains unchanged, which actually was already clear from the outset. However, since $\varkappa$ changes with velocity, also ϵ_0 has to be changed in accordance with the previous equation, and by that also the temperature of the radiation must change.

The apparent radiation gained from the work, now (after changing the velocity) has the density $\epsilon_{0,1}\tau_1$, thus the work

$$h(\epsilon_{0,1}\tau_1 - \epsilon_0\tau)$$

has to be performed for this change. (In this case, also our apparent mass introduced earlier would have another value; though no real meaning can be applied to it, as we will see in the next section.)

In the same way as before, the sign of δw plays no role here, and our considerations can be applied to finite velocity changes as well, when they are performed sufficiently slow.

In particular, if the motion of our system is completely suspended, then the gained work is equal to $h\epsilon_0\tau$, since the final value of τ (for $\beta = 0$) is zero. This work is thus equal to the surplus of the total radiation energy over the true one. The amount of the latter is thus not changing; its density remains unchanged and equal to $\epsilon_0\varkappa$. On the other hand, this radiation has now evidently a temperature, which is higher than that of the heat reservoir, with which the cavity was initially (at velocity w) connected. Because the emission capacity of the latter was (see eq. 20) $e = \frac{c}{4}\epsilon_0$; however, now we have to connect the (resting) cavity with a black body of emission capacity $e_1 = \frac{c}{4}\varkappa\epsilon_0$, so that no heat transfer takes place. Since $\varkappa > 1$, thus also $e_1 > e$, thus the temperature of radiation is increased. (According to the Stefan-Boltzmann law in the ratio $1 : \varkappa^{1/4}$)).

§ 6.

Now it is near at hand, to use this increase of temperature, in order to construct a cyclic process, which transports heat from a body of lower temperature to a body of higher temperature.

If we again imagine a system, consisting of a (black) energy reservoir of temperature T, and a cavity surrounded by mirrors of volume v. Let the system at first be at rest, and the cavity be in connection with the heat reservoir, so that the radiation energy $v\epsilon_0$ is present in the first one. If we now bring the system to velocity w, then the heat reservoir gives off the heat $v(\epsilon - \epsilon_0) = v\epsilon_0(\varkappa - 1)$ to the cavity; at the same time, the work $v\epsilon' = v\epsilon_0\tau$ must be performed. Now we separate the cavity from the heat reservoir by a mirror, and bring the velocity of our system to zero again. At that occasion, the work $v\epsilon_0\tau$ is gained from the cavity radiation again, so that no work is neither gained nor lost altogether. Yet, the density of true radiation (now the only remaining radiation in the cavity) is still equal to $\epsilon_0\varkappa$, though its temperature is now higher than T, and thus it can by itself pass to a body whose temperature is higher than T.

A new hypothesis is necessary to solve this. Such one would be the assumption, that the emission capacity of a black body changes with the velocity of translation, so that it always *caeteris paribus* is proportional to $1/\varkappa$. Then the density of the true cavity radiation would always have the value ϵ_0, and our procedure just described, to bring heat to a higher temperature, would be impossible. We must stick to the fact, that this change must be related to the true emission capacity, *i.e.*, the amount of inner energy of the radiating body which is transformed into radiation in unit time. About this quantity we always assumed until now, that it is independent of motion; in case this were incorrect, then this would also be true for our earlier considerations; thus also in our derivation of the radiation pressure. Anyway, this assumption doesn't fit into the framework of the theory presented here. Furthermore, this change also should surely concern (in the same way) the energy quantity emanated in different directions; thus the energy emanated by the elementary oscillations, *independent of the orientation of the latter*, must be changed by the same amount by motion. Exactly this seems improbable to me. Though the possibility of this hypothesis must be permitted in any case.

However, there is also another possibility for the solution of this contradiction, which is particularly remarkable by the fact, that it is the same one stated by Lorentz and Fitzgerald to explain the negative result of the experiment of Michelson and Morley. Namely the hypothesis, that the dimensions of matter depend on their absolute velocity.

The apparent contradiction with the second thermodynamic law, at which we arrived, stems indeed from the fact, that the temperature of the true cavity radiation changes at adiabatic change of velocity; or, which is the same, that its density doesn't change. This can of course be achieved by a corresponding change of volume, by which the density of the true radiation changes, so that the temperature stays the same; *i.e.*, that it assumes the value ϵ_0 especially in the case, when the cavity comes to rest again.

If we now denote (during the adiabatic velocity change) the variable density of the true radiation by ϵ , then ϵv is the momentary true energy content of the cavity. If the volume is changed, then the radiation performs a work, whose amount depends on the magnitude of the pressure. Although it would be probably no problem, to exactly calculate this work, we nevertheless want to confine ourselves to the precision, which is extended only to order β^2 (incl.). Since (as we want to preface) the considered change of volume is of order β^2, we can confine ourselves to the first term independent of β when we are stating the pressure; *i.e.*, we set the pressure equal to a third of the density of the total radiation. Since furthermore, the total and true radiation only differ by terms of order β^2 as well, we can set the work performed by the true radiation equal to

$$\frac{1}{3}\epsilon dv$$

Since the decrease of energy is equal to the performed work, we have

$$d(\epsilon v) = -\frac{1}{3}\epsilon dv.$$

or

$$vd\epsilon + \frac{4}{3}\epsilon dv = 0.$$

Our contradiction is solved, when the density of the true radiation doesn't remain constant, but (as at the isothermic change) is always equal to $\epsilon_0\varkappa$, where ϵ_0 remains constant and $\varkappa$ is the already mentioned function of the momentary velocity. Then it is the temperature which remains constant. When we insert this into the previous differential equation $\epsilon - \epsilon_0\varkappa$, and dividing ϵ_0 away by the constant, then it remains

$$vd\varkappa + \frac{4}{3}\varkappa dv = 0,$$

from which it follows:

$$v = v_0\varkappa^{-3/4}.$$

Herein, v_0 is the volume, when the velocity is equal to zero, and $\varkappa = 1$. Namely, this result is true including magnitudes of order β^2. If we insert for $\varkappa$ its value from (23), then it becomes:

$$v = v_0 \left(1 + \frac{2}{3}\beta^2\right)^{-3/4} = v_0 \left(1 - \frac{1}{2}\beta^2\right).$$

The simplest assumption is now, that for example the dimensions of matter are invariable perpendicular to their direction of motion, while the dimension coinciding with the direction of motion depends on the translation velocity by the factor $1 - \frac{1}{2}\beta^2$. The agreement with the assumption of Lorentz and Fitzgerald is thus a complete one.

I would like to allow myself, to remark that I derived this result also by another way,[23] where the knowledge of the value of the radiation pressure was not necessary; though it had to be shown in a complicated way, that the sum of the work, which must be performed at a cyclic process with moving radiating bodies at acceleration and delay, is equal to zero.

The magnitude of the apparent mass, which was calculated by us in § 5, is now modified by this assumed change of volume. However, equation (23) remains unchanged, since this modification lies outside of the limit of precision, which holds for this equations.

Vienna, July 1904.

(Received July 28, 1904).

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[1] The present treatise is an edition (partially shortened and completely new) of the content of several treatises, which were presented to the k. Akademie d. Wissenschaften in Wien (Abt IIa) in the meetings from 11. February, 28. April and 23. June 1904.

[2] P. Drude, Lehrbuch der Optik p. 447.

[3] M. Abraham, Boltzmann-Festschrift p. 87. 1904.

[4] Of course, absolute shall always be understood as "with respect to the fixed stars".

[5] It is of course not evident *a priori* as remarked by Abraham l. c. (Rem. 3) p. 251, though (as I believe) it is a very plausible assumption.

[6] M. Abraham, Ann. d. Phys. 14. p. 236. 1904. (l. c.)

[7] M. Abraham, l. c. p. 245.

[8] See H. A. Lorentz, De l'influence du mouvement de la terre sur les phénomènes lumineux. Arch. Néerl, 21. p. 106. 1886.

[9] This is given from both the electromagnetic theory (see M. Abraham, Boltzmann-Festschrift p. 81. 1904) as well as from Huyghens' principle, (see F. Hasenöhrl, Sitzungsber. d. k. Akad. d. Wissensch. zu Wien IIa 113. p. 469. 1904). If the mirror is moving into another direction, then this law is only true up to magnitudes of order β . Yet the law of reciprocity is always exactly valid for relative rays, *i.e.*, a relative ray can always traverse the same way in both directions; this is a consequence of thermodynamics (see F. Hasenöhrl, Sitzungsber. d. k. Akad. d. Wissensch. zu Wien IIa. 113. p. 493. 1904)

[10] J. H. Poynting, Phil. Trans. 202 A. 1904.

[11] Vl. v. Türin, Ann. d. Naturphil. 3. p. 270. 1904.

[12] The explanations in § 4 will confirm this.

[13] See J. H. Poynting, l. c.

[14] J. Larmor, Rep. brit. Asa. 1900.

[15] J.H. Poynting, l. c. appendix.

[16] M. Abraham, l. c. p. 257. eq. (18).

[17] F. Hasenöhrl, Sitzungsber. d. k. Akad. d. Wissensch. zu Wien IIa. Meeting from 23. June 1904.

[18] see M. Abraham, l. c. p. 252. eq. 11c).

[19] It is noticeable, that τ is connected with the magnitude $\varkappa$ given by eq. (21), is directly connected with the simple equation $\tau = \beta \frac{d\varkappa}{d\beta}$

[20] F. Hasenöhrl, Sitzungsber d. k. Akad. d. Wissensch. zu Wien, IIa. Meeting from 23. Juni 1904.

[21] See for example, M. Abraham, Ann. d. Phys. 10. p, 151. 1903.

[22] The index 1 shall have the same meaning as earlier.

[23] F. Hasenöhrl, Sitzungsber. d. k. Akad. d. Wissensch. zu Wien IIa. p. 469. 1904.

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