Chapter 2

Space and Time

Gentlemen! The views of space and time which I want to present to you arose from the domain of experimental physics, and therein lies their strength. Their tendency is radical. From now onwards space by itself and time by itself will recede completely to become mere shadows and only a type of union of the two will still stand independently on its own.

I.

I want to show first how to move from the currently adopted mechanics through purely mathematical reasoning to modified ideas about space and time. The equations of Newtonian mechanics show a twofold invariance. First, their form is preserved when subjecting the specified spatial coordinate system to *any change of position*; second, when it changes its state of motion, namely when any *uniform translation* is impressed upon it; also, the zero point of time plays no role. When one feels ready for the axioms of mechanics, one is accustomed to regard the axioms of geometry as settled and probably for this reason those two invariances are rarely mentioned in the same breath. Each of them represents a certain group of transformations for the differential equations of mechanics. The existence of the first group can be seen as reflecting a fundamental characteristic of space. One always tends to treat the second group with disdain in order to unburden one's mind that one can never determine from physical phenomena whether space, which is assumed to be at rest, may not after all be in uniform translation. Thus these two groups lead completely separate lives side by side. Their entirely heterogeneous character may have discouraged any intention to compose them. But it is the composed complete group as a whole that gives us to think.

We will attempt to visualize the situation graphically. Let x, y, z be orthogonal coordinates for space and let t denote time. The objects of our perception are always connected to places and times. No one has noticed a place other than at a time and a time other than at a place. However I still respect the dogma that space and time each have an independent meaning. I will call a point in space at a given time, i.e. a system of values x, y, z, t a *worldpoint*. The manifold of all possible systems of values x, y, z, t will be called the *world*. With a hardy piece of chalk I can draw four world axes on the blackboard. Even *one* drawn axis consists of nothing but vibrating molecules and also makes the journey with the Earth in the Universe, which already requires sufficient abstraction; the somewhat greater abstraction associated with the number 4 does not hurt the mathematician. To never let a yawning emptiness, let us imagine that everywhere and at any time something perceivable exists. In order not to say matter or electricity I will use the word substance for that thing. We focus our attention on the substantial point existing at the worldpoint x, y, z, t and imagine that we can recognize this substantial point at any other time. A time element dt may correspond to the changes dx, dy, dz of the spatial coordinates of this substantial point. We then get an image, so to say, of the eternal course of life of the substantial point, a curve in the world, a *worldline*, whose points can be clearly related to the parameter t from $-\infty$ to $+\infty$. The whole world presents itself as resolved into such worldlines, and I want to say in advance, that in my understanding the laws of physics can find their most complete expression as interrelations between these worldlines.

Through the concepts of space and time the x, y, z-manifold $t = 0$ and its two sides $t > 0$ and $t < 0$ fall apart. If for simplicity we hold the chosen origin of space and time fixed, then the first mentioned group of mechanics means that we can subject the x, y, z-axes at $t = 0$ to an arbitrary rotation about the origin corresponding to the homogeneous linear transformations of the expression

$$x^2 + y^2 + z^2.$$

The second group, however, indicates that, also without altering the expressions of the laws of mechanics, we may replace

$$x, y, z, t \quad \text{by} \quad x - \alpha t, \ y - \beta t, \ z - \gamma t, \ t,$$

where α, β, γ are any constants. The time axis can then be given a completely arbitrary direction in the upper half of the world $t > 0$. What has

now the requirement of orthogonality in space to do with this complete freedom of choice of the direction of the time axis upwards?

To establish the connection we take a positive parameter c and look at the structure
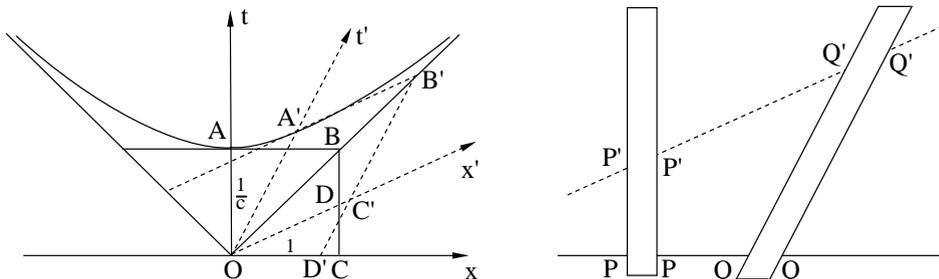
$$c^2t^2 - x^2 - y^2 - z^2 = 1.$$

Fig. 1

It consists of two sheets separated by $t = 0$ by analogy with a two-sheeted hyperboloid. We consider the sheet in the region $t > 0$ and we will now take those homogeneous linear transformations of x, y, z, t in four new variables x', y', z', t' so that the expression of this sheet in the new variables has the same form. Obviously, the rotations of space about the origin belong to these transformations. A full understanding of the rest of those transformations can be obtained by considering such among them for which y and z remain unchanged. We draw (Fig. 1) the intersection of that sheet with the plane of the x- and the t-axis, i.e. the upper branch of the hyperbola $c^2t^2 - x^2 = 1$ with its asymptotes. Further we draw from the origin O an arbitrary radius vector OA' of this branch of the hyperbola; then we add the tangent to the hyperbola at A' to intersects the right asymptote at B'; from $OA'B'$ we complete the parallelogram $OA'B'C'$; finally, as we will need it later, we extend $B'C'$ so that it intersects the x-axis at D'. If we now regard OC' and OA' as axes for new coordinates x', t', with the scale units $OC' = 1$, $OA' = 1/c$, then that branch of the hyperbola again obtains the expression $ct'^2 - x'^2 = 1$, $t' > 0$, and the transition from x, y, z, t to x', y', z', t' is one of the transformations in question. These transformations plus the arbitrary displacements of the origin of space and time constitute a group of transformations which still depends on the parameter c and which I will call G_c.

If we now increase c to infinity, so $1/c$ converges to zero, it is clear from the figure that the branch of the hyperbola leans more and more towards the x-axis, that the angle between the asymptotes becomes greater, and in the limit that special transformation converts to one where the t'-axis may be in any upward direction and x' approaches x ever more closely. By taking this into account it becomes clear that the group G_c in the limit $c = \infty$, that is the group G_∞, is exactly the complete group which is associated with the Newtonian mechanics. In this situation, and since G_c is mathematically more understandable than G_∞, there could have probably been a mathematician with a free imagination who could have come up with the idea that at the end natural phenomena do not actually possess an invariance with the group G_∞, but rather with a group G_c with a certain finite c, which is *extremely great* only in the ordinary units of measurement. Such an insight would have been an extraordinary triumph for pure mathematics. Now mathematics expressed only staircase wit here, but it has the satisfaction that, due to its happy antecedents with their senses sharpened by their free and penetrating imagination, it can grasp the profound consequences of such remodelling of our view of nature.

I want to make it quite clear what the value of c will be with which we will be finally dealing. c is the *velocity of the propagation of light in empty space.* To speak neither of space nor of emptiness, we can identify this magnitude with the ratio of the electromagnetic to the electrostatic unit of the quantity of electricity.

The existence of the invariance of the laws of nature with respect to the group G_c would now be stated as follows:

From the entirety of natural phenomena, through successively enhanced approximations, it is possible to deduce more precisely a reference system x, y, z, t, space and time, by means of which these phenomena can be then represented according to certain laws. But this reference system is by no means unambiguously determined by the phenomena. *One can still change the reference system according to the transformations of the above group G_c arbitrarily without changing the expression of the laws of nature in the process.*

For example, according to the figure depicted above one can call t' time, but then must necessarily, in connection with this, define space by the manifold of three parameters x', y, z in which the laws of physics would then have exactly the same expressions by means of x', y, z, t' as by means of x, y, z, t. Hereafter we would then have in the world no more *the* space, but an infinite number of spaces analogously as there is an infinite number of planes in three-dimensional space. Three-dimensional geometry becomes a chapter

in four-dimensional physics. You see why I said at the beginning that space and time will recede completely to become mere shadows and only a world in itself will exist.

II.

Now the question is, what circumstances force us to the changed view of space and time, does it actually never contradict the phenomena, and finally, does it provide advantages for the description of the phenomena?

Before we discuss these questions, an important remark is necessary. Having individualized space and time in some way, a straight worldline parallel to the t-axis corresponds to a stationary substantial point, a straight line inclined to the t-axis corresponds to a uniformly moving substantial point, a somewhat curved worldline corresponds to a non-uniformly moving substantial point. If at any worldpoint x, y, z, t there is a worldline passing through it and we find it parallel to any radius vector OA' of the previously mentioned hyperboloidal sheet, we may introduce OA' as a new time axis, and with the thus given new concepts of space and time, the substance at the worldpoint in question appears to be at rest. We now want to introduce this fundamental axiom:

With appropriate setting of space and time the substance existing at any worldpoint can always be regarded as being at rest.

This axiom means that at every worldpoint[1] the expression

$$c^2 \mathrm{d}t^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2$$

is always positive, which is equivalent to saying that any velocity v is always smaller than c. Then c would be an upper limit for all substantial velocities and that is precisely the deeper meaning of the quantity c. In this understanding the axiom is at first glance slightly displeasing. It should be noted, however, that a modified mechanics, in which the square root of that second order differential expression enters, is now gaining ground, so that cases with superluminal velocity will play only such a role as that of figures with imaginary coordinates in geometry.

The *impulse* and true motivation for *accepting the group G_c* came from noticing that the differential equation for the propagation of light waves in the empty space possesses that group G_c[2]. On the other hand, the concept of

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[1] *Editor's note:* Minkowski means at every worldpoint along the worldline of the substance.

[2] An important application of this fact can already be found in W. Voigt, Göttinger Nachrichten, 1887, S. 41.

a rigid body has meaning only in a mechanics with the group G_∞. If one has optics with G_c, and if, on the other hand, there were rigid bodies, it is easy to see that *one t-direction* would be distinguished by the two hyperboloidal sheets corresponding to G_c and G_∞, and would have the further consequence that one would be able, by using appropriate rigid optical instruments in the laboratory, to detect a change of phenomena at various orientations with respect to the direction of the Earth's motion. All efforts directed towards this goal, especially a famous interference experiment of Michelson had, however, a negative result. To obtain an explanation, H. A. Lorentz made a hypothesis, whose success lies precisely in the invariance of optics with respect to the group G_c. According to Lorentz every body moving at a velocity v must experience a reduction in the direction of its motion namely in the ratio

$$1 : \sqrt{1 - \frac{v^2}{c^2}}.$$

This hypothesis sounds extremely fantastical. Because the contraction is not to be thought of as a consequence of resistances in the ether, but merely as a gift from above, as an accompanying circumstance of the fact of motion.

I now want to show on our figure that the Lorentzian hypothesis is completely equivalent to the new concept of space and time, which makes it much easier to understand. If for simplicity we ignore y and z and think of a world of one spatial dimension, then two strips, one upright parallel to the t-axis and the other inclined to the t-axis (see Fig. 1), are images for the progression in time of a body at rest and a body moving uniformly, where each preserves a constant spatial dimension. OA' is parallel to the second strip, so we can introduce t' as time and x' as a space coordinate and then it appears that the second body is at rest, whereas the first – in uniform motion. We now assume that the first body has length l when considered at rest, that is, the cross section PP of the first strip and the x-axis is equal to $l \cdot OC$, where OC is the measuring unit on the x-axis, and, on the other hand, that the second body has the same length l when *regarded at rest*; then the latter means that the cross-section of the second strip *measured parallel to the x'-axis* is $Q'Q' = l \cdot OC'$. We have now in these two bodies images of two *equal* Lorentz electrons, one stationary and one uniformly moving. But if we go back to the original coordinates x, t, we should take as the dimension of the second electron the cross section QQ of its associated strip *parallel to the x-axis*. Now as $Q'Q' = l \cdot OC'$, it is obvious that $QQ = l \cdot OD'$. If dx/dt for the second strip is $= v$, an easy calculation

gives $OD' = OC \cdot \sqrt{1 - \frac{v^2}{c^2}}$, therefore also $PP : QQ = 1 : \sqrt{1 - \frac{v^2}{c^2}}$. This is the meaning of the Lorentzian hypothesis of the contraction of electrons in motion. Regarding, on the other hand, the second electron as being at rest, that is, adopting the reference system x', t', the length of the first electron will be the cross section $P'P'$ of its strip parallel to OC', and we would find the first electron shortened with respect to the second in exactly the same proportion; from the figure we also see that

$$P'P' : Q'Q' = OD : OC' = OD' : OC = QQ : PP.$$

Lorentz called t', which is a combination of x and t, *local time* of the uniformly moving electron, and associated a physical construction with this concept for a better understanding of the contraction hypothesis. However, it is to the credit of A. Einstein[3] who first realized clearly that the time of one of the electrons is as good as that of the other, i.e. that t and t' should be treated equally. With this, time was deposed from its status as a concept unambiguously determined by the phenomena. The concept of space was shaken neither by Einstein nor by Lorentz, maybe because in the above-mentioned special transformation, where the plane of x', t' coincides with the plane x, t, an interpretation is possible as if the x-axis of space preserved its position. To step over the concept of space in such a way is an instance of what can be achieved only due to the audacity of mathematical culture. After this further step, which is indispensable for the true understanding of the group G_c, I think the word *relativity postulate* used for the requirement of invariance under the group G_c is very feeble. Since the meaning of the postulate is that through the phenomena only the four-dimensional world in space and time is given, but the projection in space and in time can still be made with a certain freedom, I want to give this affirmation rather the name *the postulate of the absolute world* (or shortly the world postulate).

III.

Through the world postulate an identical treatment of the four identifying quantities x, y, z, t becomes possible. I want to explain now how, as a result of this, we gain more understanding of the forms under which the laws of physics present themselves. Especially the concept of *acceleration* acquires a sharply prominent character.

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[3]A. Einstein, Annalen der Physik 17 (1905), S. 891; Jahrbuch der Radioaktivität und ELektronik 4 (1907), S. 411.

I will use a geometric way of expression, which presents itself immediately when one implicitly ignores z in the triple x, y, z. An arbitrary worldpoint O can be taken as the origin of space-time. The *cone*

$$c^2t^2 - x^2 - y^2 - z^2 = 0$$

with O as the apex (Fig. 2) consists of two parts, one with values $t < 0$, the other one with values $t > 0$.
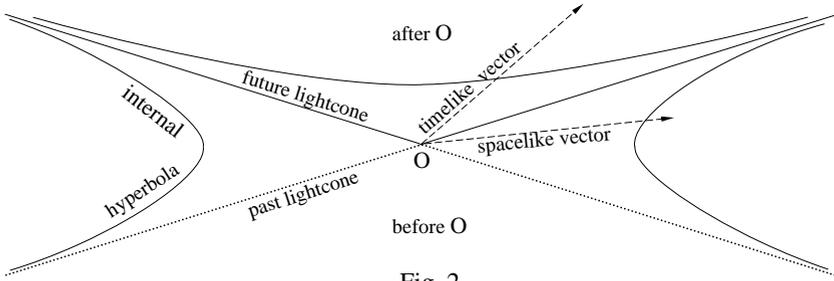

Fig. 2

The first, the *past lightcone of O*, consists, we can say, of all worldpoints which "send light to O", the second, the *future lightcone of O*, consists of all worldpoints which "receive light from O"[4]. The area bounded solely by the past lightcone may be called *before O*, whereas the area bounded solely by the future lightcone – *after O*. Situated after O is the already considered hyperboloidal sheet

$$F = c^2t^2 - x^2 - y^2 - z^2 = 1, \ t > 0$$

The area *between the cones* is filled with the one-sheeted hyperboloidal structures

$$-F = x^2 + y^2 + z^2 - c^2t^2 = k^2$$

for all constant positive values of k^2. Essential for us are the hyperbolas with O as the center, located on the latter structures. The individual branches of these hyperbolas may be briefly called *internal hyperbolas with center O*.

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[4]*Editor's and translator's note:* I decided to translate the words *Vorkegel* and *Nachkegel* as *past lightcone* and *future lightcone*, respectively, for two reasons. First, this translation reflects the essence of Minkowski's idea – (i) all worldpoints on the past lightcone "send *light* to O", which means that they all can influence O and therefore lie in the past of O; (ii) all worldpoints on the future lightcone "receive *light* from O", which means that they all can be influenced by O and therefore lie in the *future* of O. Second, the terms *past lightcone* and *future lightcone* are now widely accepted in spacetime physics.

Such a hyperbola would be thought of as the worldline of a substantive point, which represents its motion that increases asymptotically to the velocity of light c for $t = -\infty$ and $t = +\infty$.

If we now call, by analogy with vectors in space, a directed line in the manifold x, y, z, t a vector, we have to distinguish between the *timelike* vectors with directions from O to the sheet $+F = 1, t > 0$, and the *spacelike* vectors with directions from O to $-F = 1$. The time axis can be parallel to any vector of the first kind. Every worldpoint between the future lightcone and the past lightcone of O can be regarded, by a choice of the reference system, as *simultaneous* with O as well as *earlier* than O or *later* than O. Each worldpoint within the past lightcone of O is necessarily always earlier than O, each worldpoint within the future lightcone is necessarily always later than O. The transition to the limit $c = \infty$ would correspond to a complete folding of the wedge-shaped section between the cones into the flat manifold $t = 0$. In the figures this section is intentionally made with different widths.

We decompose any vector, such as that from O to x, y, z, t into four *components* x, y, z, t. If the directions of two vectors are, respectively, that of a radius vector OR from O to one of the surfaces $\mp F = 1$, and that of a tangent RS at the point R on the same surface, the vectors are called *normal* to each other. Accordingly,

$$c^2 t t_1 - x x_1 - y y_1 - z z_1 = 0$$

is the condition for the vectors with components x, y, z, t and x_1, y_1, z_1, t_1 to be normal to each other.

The *measuring units* for the *magnitudes* of vectors in different directions may be fixed by assigning to a spacelike vector from O to $-F = 1$ always the magnitude 1, and to a timelike vector from O to $+F = 1, t > 0$ always the magnitude $1/c$.

Let us now imagine a worldpoint $P(x, y, z, t)$ through which the worldline of a substantial point is passing, then the magnitude of the timelike vector dx, dy, dz, dt along the line will be

$$d\tau = \frac{1}{c}\sqrt{c^2 dt^2 - dx^2 - dy^2 - dz^2}.$$

The integral $\int d\tau = \tau$ of this magnitude, taken along the worldline from any fixed starting point P_0 to the variable end point P, we call the *proper time* of the substantial point at P. On the worldline we consider x, y, z, t, i.e. the components of the vector OP, as functions of the proper time τ; denote their first derivatives with respect to τ by $\dot{x}, \dot{y}, \dot{z}, \dot{t}$; their second derivatives with

respect to τ by $\ddot{x}, \ddot{y}, \ddot{z}, \ddot{t}$, and call the corresponding vectors, the derivative of the vector OP with respect to τ the *velocity vector at P* and the derivative of the velocity vector with respect to τ the *acceleration vector at P*. As

$$c^2\dot{t}^2 - \dot{x}^2 - \dot{y}^2 - \dot{z}^2 = c^2$$

it follows that

$$c^2\dot{t}\ddot{t} - \dot{x}\ddot{x} - \dot{y}\ddot{y} - \dot{z}\ddot{z} = 0,$$

i.e. the velocity vector is the timelike vector of magnitude 1 in the direction of the worldline at P, and the acceleration vector at P is normal to the velocity vector at P, so it is certainly a spacelike vector.
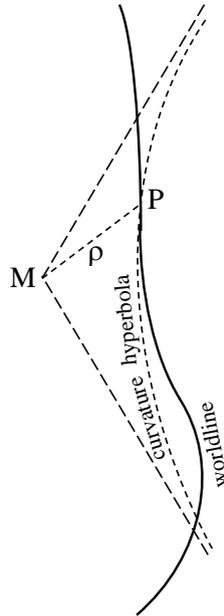

Fig. 3

Now there is, as is easily seen, a specific branch of the hyperbola, which has three infinitely adjacent points in common with the worldline at P, and whose asymptotes are generators of a past lightcone and a future lightcone (see Fig. 3). This branch of the hyperbola will be called the *curvature hyperbola* at P. If M is the center of this hyperbola, we have here an internal hyperbola with center M. Let ρ be the magnitude of the vector MP, *so we recognize the acceleration vector at P as the vector in the direction MP of magnitude c^2/ρ.*

If $\ddot{x}, \ddot{y}, \ddot{z}, \ddot{t}$ are all zero, the curvature hyperbola reduces to the straight line touching the worldline at P, and we should set $\rho = \infty$.

<div align="center">IV.</div>

To demonstrate that the adoption of the group G_c for the laws of physics never leads to a contradiction, it is inevitable to undertake a revision of all physics based on the assumption of this group. This revision has been done successfully to some extent for questions of thermodynamics and heat radiation[5], for the electromagnetic processes, and finally, with the retention of the concept of mass, for mechanics.[6]

For the latter domain, the question that should be raised above all is: When a force with the spatial components X, Y, Z acts at a worldpoint $P(x, y, z, t)$, where the velocity vector is $\dot{x}, \dot{y}, \dot{z}, \dot{t}$, as what force this force should be interpreted for any change of the reference system? Now there exist some proven approaches to the ponderomotive force in the electromagnetic field in cases where the group G_c is undoubtedly permissible. These approaches lead to the simple rule: *When the reference system is changed, the given force transforms into a force in the new space coordinates in such a way that the corresponding vector with the components*

$$\dot{t}X, \ \dot{t}Y, \ \dot{t}Z, \ \dot{t}T$$

remains unchanged, and where

$$T = \frac{1}{c^2}\left(\frac{\dot{x}}{\dot{t}}X + \frac{\dot{y}}{\dot{t}}Y + \frac{\dot{z}}{\dot{t}}Z\right)$$

is the work done by the force at the worldpoint divided by c^2. This vector is always normal to the velocity vector at P. Such a force vector, representing a force at P, will be called a *motive force vector* at P.

Now let the worldline passing through P represent a substantial point with constant *mechanical mass m*. The multiplied by m velocity vector at P will be called the *momentum vector at P*, and the multiplied by m acceleration vector at P will be called the *force vector of the motion at P*.

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[5]M. Planck, "Zur Dynamik bewegter Systeme," Sitzungsberichte der k. preußischen Akademie der Wissenschaften zu Berlin, 1907, S. 542 (auch Annalen der Physik, Bd. 26, 1908, S. 1).

[6]H. Minkowski, "Die Grundgleichungen für die elektromagnetischen Vorgänge in bewegten Körpern", Nachrichten der k. Gesellschaft der Wissenschaft zu Göttingen, mathematisch-physikalische Klasse, 1908, S. 53 und Mathematische Annalen, Bd. 68, 1910, S. 527

According to these definitions, the law of motion for a point mass with a given force vector is:[7]

The force vector of the motion is equal to the motive force vector.

This assertion summarizes four equations for the components for the four axes, wherein the fourth can be regarded as a consequence of the first three because both vectors are from the start normal to the velocity vector. According to the above meaning of T, the fourth equation is undoubtedly the law of conservation of energy. The *kinetic energy* of the point mass is defined as the *component of the momentum vector along the t-axis multiplied by c^2*. The expression for this is

$$mc^2 \frac{dt}{d\tau} = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}},$$

which is, the expression $\frac{1}{2}mv^2$ of Newtonian mechanics after the subtraction of the additive constant term mc^2 and neglecting magnitudes of the order $1/c^2$. The *dependence of the energy on the reference system* is manifested very clearly here. But since the t-axis can be placed in the direction of each timelike vector, then, on the other hand, the law of conservation of energy, formed for every possible reference system, already contains the whole system of the equations of motion. In the discussed limiting case $c = \infty$, this fact will retain its importance for the axiomatic structure of Newtonian mechanics and in this sense has been already noticed by J. R. Schütz[8]

From the beginning we can determine the ratio of the units of length and time in such a way that the natural limit of velocity becomes $c = 1$. If we introduce $\sqrt{-1}t = s$ instead of t, then the quadratic differential expression

$$d\tau^2 = -dx^2 - dy^2 - dz^2 - ds^2$$

becomes completely symmetric in x, y, z, s and this symmetry is carried over to any law that does not contradict the world postulate. Thus the essence of this postulate can be expressed mathematically very concisely in the mystical formula:

$$3 \cdot 10^5 \; km = \sqrt{-1} \; seconds.$$

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[7]H. Minkowski, loc. cit., p. 107. Cf. also M. Planck, Verhandlungen der Physikalischen Gesellschaft, Bd. 4, 1906, S. 136.

[8]J. R. Schütz, "Das Prinzip der absoluten Erhaltung der Energie", Nachrichten der k. Gesellschaft der Wissenschaften zu Göttingen, mathematisch-physikalische Klasse, 1897, S. 110.

V.

The advantages resulting from the world postulate may most strikingly be proved by indicating the effects from *an arbitrarily moving point charge* according to the Maxwell-Lorentz theory. Let us imagine the worldline of such a pointlike electron with charge e, and take on it the proper time τ from any initial point. To determine the field induced by the electron at any worldpoint P_1 we construct the past lightcone corresponding to P_1 (Fig. 4). It intersects the infinite worldline of the electron obviously at a single point P because the tangents to every point on the worldline are all timelike vectors. At P we draw the tangent to the worldline and through P_1 construct the normal $P_1 Q$ to this tangent. Let the magnitude of $P_1 Q$ be r. According to the definition of a past lightcone the magnitude of PQ should be r/c. *Now the vector of magnitude e/r in the direction PQ represents through its components along the x-, y-, z-axes, the vector potential multiplied by c, and through the component along the t-axis, the scalar potential of the field produced by e at the worldpoint P_1.* This is the essence of the elementary laws formulated by A. Liénard and E. Wiechert.[9]
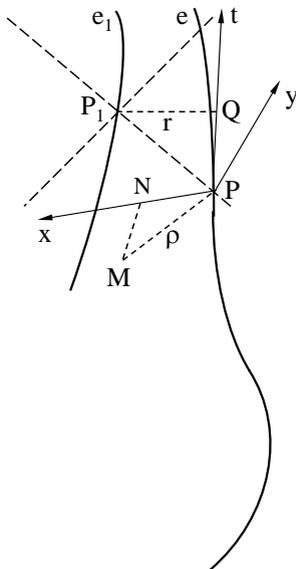

Fig. 4

[9]A. Liénard, "Champ électrique et magnétique produit par une charge concentré en un point et animée d'un mouvement quelconque", L'Éclairage électrique, T. 16, 1898, pp. 5, 53, 106; E. Wiechert, "Elektrodynamische Elementargesetze", Archives Néerlandaiaes des Sciences exactes et naturelles (2), T. 5, 1900, S. 549.

Then it emerges in the description itself of the field caused by the electron that the division of the field into electric and magnetic forces is a relative one with respect to the specified time axis; most clearly the two forces considered together can be described in some, though not complete, analogy with the wrench in mechanics. I now want to describe *the ponderomotive action of an arbitrarily moving point charge exerted on another arbitrarily moving point charge.* Let us imagine that the worldline of a second pointlike electron of charge e_1 goes through the worldpoint P_1. We define P, Q, r as before, then construct (Fig. 4) the center M of the curvature hyperbola at P, and finally the normal MN from M to an imagined straight line from P parallel to QP_1. We now fix a reference system with its origin at P in the following way: the t-axis in the direction of PQ, the x-axis in the direction of QP_1, the y-axis in the direction of MN, and lastly the direction of the z-axis is determined as being normal to the t-, x-, y-axes. Let the acceleration vector at P be $\ddot{x}, \ddot{y}, \ddot{z}, \ddot{t}$, the velocity vector at P_1 be $\dot{x}_1, \dot{y}_1, \dot{z}_1, \dot{t}_1$. *Now the motive force vector exerted by the first arbitrarily moving electron e on the second arbitrarily moving electron e_1 at P_1 will be*

$$-ee_1(\dot{t}_1 - \frac{\dot{x}_1}{c})\mathfrak{K}$$

where for the components $\mathfrak{K}_x, \mathfrak{K}_y, \mathfrak{K}_z, \mathfrak{K}_t$ of the vector $\mathfrak{K}$ three relations exist:

$$c\mathfrak{K}_t - \mathfrak{K}_x = \frac{1}{r^2}, \quad \mathfrak{K}_y = \frac{\ddot{y}}{c^2 r}, \quad \mathfrak{K}_z = 0$$

and fourthly this vector $\mathfrak{K}$ is normal to the velocity vector at P_1, and this circumstance alone makes it dependent on the latter velocity vector.

If we compare this assertion with the previous formulations[10] of the same elementary law of the ponderomotive action of moving point charges on one another, we are compelled to admit that the relations considered here reveal their inner being in full simplicity only in four dimensions, whereas on a three dimensional space, forced upon us from the beginning, they cast only a very tangled projection.

In mechanics reformed in accordance with the world postulate, the disturbing disharmony between Newtonian mechanics and the modern electrodynamics disappears by itself. In addition, I want to touch on the status of the *Newtonian law of attraction* with respect to this postulate. I will consider two point masses m, m_1, represented by their worldlines, and that

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[10]K. Schwazschild, Nachrichten der k. Gesellschaft der Wissenschaften zu Göttinger, mathematisch-physikalische Klasse, 1903, S. 132; H. A. Lorentz, Enzyklopädie der mathematischen Wissenschaften, V, Art. 14, S. 199.

m exerts a motive force vector on m_1 exactly as in the case of electrons, except that instead of $-ee_1$ $+mm_1$ should be used. We can now specifically consider the case when the acceleration vector of m is constantly zero, then we may choose t in such a way that m is regarded as at rest, and assume that only m_1 move under the motive force vector which originates from m. If we now modify this specified vector by adding the factor $\dot{t}^{-1} = \sqrt{1 - \frac{v^2}{c^2}}$, which up to magnitudes of the order $1/c^2$ is equal to 1, it can be seen[11] that for the positions x_1, y_1, z_1 of m_1 and their progression in time, we arrive exactly at Kepler's laws, except that instead of the times t_1 the proper times τ_1 of m_1 should be used. On the basis of this simple remark we can then see that the proposed law of attraction associated with the new mechanics is no less well suited to explain the astronomical observations than the Newtonian law of attraction associated with the Newtonian mechanics.

The fundamental equations for the electromagnetic processes in ponderable bodies are entirely in accordance with the world postulate. Actually, as I will show elsewhere, there is no need to abandon the derivation of these equations which is based on ideas of the electron theory as taught by Lorentz.

The validity without exception of the world postulate is, I would think, the true core of an electromagnetic world view which, as Lorentz found it and Einstein further unveiled it, lies downright and completely exposed before us as clear as daylight. With the development of the mathematical consequences of this postulate, sufficient findings of its experimental validity will be arrived at so that even those to whom it seems unsympathetic or painful to abandon the prevailing views become reconciled through the thought of a pre-stabilized harmony between mathematics and physics.

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[11]H. Minkowski, loc. cit., p. 110.