Einstein’s 1934 two-blackboard derivation of energy-mass equivalence

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(Received 25 July 2006; accepted 9 July 2007)

We use a famous and a rare picture of Einstein to reconstruct the context of a lecture he gave on the derivation of the equivalence of energy and mass in Pittsburgh in 1934. This lecture is interesting from a historical and sociological point of view because, at the time, Einstein was at the height of his fame, the equivalence of energy and mass was being discussed in newspapers, and his presence in Pittsburgh created much attention among the general public. Einstein exhibited his well-known intuitive style of using only the most important physical information in the zero-momentum frame derivation. His method was simple and direct and is relevant to those who teach the zero-momentum frame idea. From the perspective of the nonspecialists in the Pittsburgh audience, it was presented at an expert level without allowing for many explanatory concessions we would take for granted today. A definitive picture of Einstein, in front of his famous energy equation, was missed by photographers who posed him with the wrong blackboard in the background. © 2007 American Association of Physics Teachers.

[DOI: 10.1119/1.2772277]

Photographers were most fond of snapping pictures of Albert Einstein standing before a blackboard containing equations, thus reinforcing the stereotype of the ethereal scientist amid his remote and abstract mathematical world. There are abundant pictures fitting this format, one of which we analyzed earlier. Most are obviously posed shots taken either before or after a lecture (see Fig. 1); only a few are candid, taken during the lecture (see Fig. 2).

Both pictures were taken at the Carnegie Institute of Technology (now Carnegie-Mellon University) in Pittsburgh, Pennsylvania on 28 December 1934. Einstein was delivering the prestigious Josiah Willard Gibbs Lecture of the American Mathematical Society (AMS). Figure 2 was taken from the balcony early in the lecture. The logistics of making a blackboard presentation to over 400 people were daunting in 1934, and special electrical lighting fixed to the top of two blackboards was devised. Prior to the talk the two illuminated blackboards were filled with the requisite equations—the “hieroglyphics of higher mathematics” in the words of one newspaper reporter. We see Einstein in Fig. 2 caught in the act of gesturing with his left hand as he explains something about the equations near the top of the left blackboard.

In the popular mind the ideal picture would have been one with the equation $E=mc^2$ emblazoned on the blackboard. The title of Einstein’s lecture was, after all, “An elementary proof of the theorem concerning the equivalence of mass and energy.” Not surprisingly, the illustrious equation is on one of the blackboards. Unfortunately it is not in Fig. 1, the close-up shot taken before the lecture, which depicts Einstein with the left blackboard of Fig. 2. Instead, it is the right blackboard that contains the equation. But its format may disappoint or confuse the average viewer, because from the start of the lecture Einstein employed the convention of setting the speed of light $c$ to unity. Hence a close look at the lower left section of the right blackboard in Fig. 2 reveals the relation $\Delta E_0 = \Delta mc$, and below it is $E_0 = mc$. As far as we know Fig. 2 is the only extant picture with Einstein and his famous equation.

At this time in his life Einstein was a recent immigrant, having accepted an offer from the newly formed Institute for Advanced Study in Princeton, New Jersey. He was an immigrant, but not a stranger to the United States, having visited and sojourned several times before. In 1921 he went on a tour to support the Zionist movement and to raise funds for the planned Hebrew University of Jerusalem. His visit included lectures on relativity in New York City, Chicago, Boston, and Princeton. In the early 1930s he spent several winters in California at Caltech, brought by Robert A. Millikan, who hoped eventually to convince Einstein to take a permanent position there. But the new Institute in Princeton was also pursuing him. During his third winter sojourn at Caltech (1932–1933) Hitler came to power and Einstein remarked to his wife, Elsa, that they would never return to Germany. After an interregnum in Belgium, England, and Switzerland, with Einstein lecturing at several universities, Einstein and his wife arrived in America in October of 1933 and settled into Princeton. The Institute had trumped Millikan in snagging Einstein, who remained at Princeton until his death in 1955. The 1934 trip to Pittsburgh was one of his first public appearances in his newly adopted country.

The Pittsburgh meeting of the AMS was held from Friday, 28 December 1934 to Tuesday, 1 January 1935. Einstein arrived by train from Princeton on Thursday; he delivered his lecture late Friday afternoon. He attended a dinner in his honor Saturday and left Sunday, taking the train back to Princeton. He did not stay for the entire meeting and, as far as we know, did not attend any sessions.

A friend, Leon L. Watters, whom he had met at Caltech and who now lived in New York City, accompanied him on the trip. Their families often vacationed together. At the time of Einstein’s departure Elsa was running a very high fever, reported as 104 °F, but Einstein felt he could not disappoint the mathematicians who had honored him with this prestigious lecture. Watters escorted Einstein at Elsa’s request, to make sure that Einstein took care of himself (namely, to get plenty of rest and especially to refrain from smoking his...
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to occur for Einstein on Friday morning—at which time,
at the home of a friend, where he arranged a news conference

identified as Robertson. In a ten-page report Robertson made
the editor sent it to an anonymous referee, who has now been
written by his collaborator Nathan Rosen and himself, and

1936 Einstein sent the journal a manuscript on gravity waves

and was accepted by Physical Review.

Appropriately a cosmology question was also raised at the
news conference. During his first visit to Caltech (1930–
1931), Einstein met Edwin Hubble, who had recently published
what was to become his law of the recession of the

galaxies (although he did not necessarily interpret it that way);
nonetheless, it convinced Einstein that he had been

wrong in 1917 in assuming a static universe. As Einstein put

it in Pittsburgh: The ensuing question was whether the uni-

verse was either finite (closed) or infinite (open); the former

entailed a positive curvature of space, whereas the latter a

negative one. He again admitted that he was wrong in assum-
ing a static universe, and he pointed to the new 200 in.
telescope at Mount Palomar for possibly deciding this question
by measuring the curvature of space.

Finally among the questions asked was an obvious one on
atomic energy. The Gibbs Lecture was delivered at an auspi-
cious time, because there was talk in the popular press that
Einstein’s simple equation \(E=mc^2\) could be applied to har-
ness the power of the atom (really the nucleus). In light of

the topic of his lecture—and, of course, the subsequent his-
tory over the next decade—Einstein’s reply is ironic: he did

not believe it was possible. “I am not a prophet, but I feel

absolutely (or at least nearly) sure that it will not be possible
to convert matter into energy for practical purposes. You

must employ a lot of energy to get any energy out of the
molecule, and the rest is lost.” He ended with an analogy:
“It’s like shooting birds in the dark in a country where there
are only a few birds.”

The last line was quoted in subse-
quent articles in newspapers across the country: the title in a
Pittsburgh paper read, “Atom energy hope is spiked by Ein-
stein,” and in another, “Man can’t harness atom as source of
energy [Einstein] says.”

Other famous physicists were in Pittsburgh for the confer-
ence, such as Millikan, Arthur Compton, and Harold Urey,
but Einstein dominated the headlines. He was front-page
news during his visit; even so, he and the conference were
competing for front-page space with another matter of con-
siderable public attraction—the ongoing Lindbergh baby
trial.

At Einstein’s request, the Gibbs Lecture was not held in a
large hall for all to attend. Instead it was confined to a select
crowd of about 400–450 in a theater at Carnegie Tech. Mem-
bers of the AMS received about 200 tickets; about 100 were
distributed by lot; and another 100 or so went to selected
individuals. There were up to 3000 people trying to get into

pipe!). Their roles, however, reversed as Watters developed a
heavy cough and Einstein watched over him.

During his three nights in Pittsburgh Einstein was given a
suite in the house of Edgar J. Kaufmann, owner of a major
department store in the city. In 1936 Kaufmann com-
misioned the architect Frank Lloyd Wright to design a house for
him in Bear Run near Pittsburgh. Photographs of the famous
Kaufmann house, better known as “Fallingwater,” are

found in most art and architecture textbooks. Watters stayed
at the home of a friend, where he arranged a news conference
to occur for Einstein on Friday morning—at which time,

according to Watters, Einstein arrived “in a jolly mood.”

At the news conference was the mathematical physicist Howard
Percy Robertson, who also had come from Princeton with
Einstein, and who later played a role in a dispute Einstein
had with Physical Review. (Here’s the story, in brief. In June
1936 Einstein sent the journal a manuscript on gravity waves
written by his collaborator Nathan Rosen and himself, and

the editor sent it to an anonymous referee, who has now been
identified as Robertson. In a ten-page report Robertson made
some critical comments and pointed to a possible error in the
manuscript. Einstein, apparently expecting immediate publi-
cation, was furious and withdrew the manuscript, never

sending another manuscript to the journal for rest of his life.

At least one reporter at the 1934 news conference was
knowledgeable about Einstein’s work. This reporter quizzed
him on his progress toward a unified field theory, to which
Einstein replied that he hoped to solve it although “The prob-
ability of finding an answer is very small.” Likewise, Ein-

stein was questioned on the breakdown of classical determin-
ism in quantum physics. Einstein asserted that, at present,
there was no answer. He said: “In former days it was the

belief of scientists that … if you are given the initial state of
a system, and observe it, and if you know the laws of nature,
then you could predict the state of the system in the future.
But [Werner] Heisenberg has shown that this is not the case,
for we cannot observe the initial state without influencing
the system in a way unknown to us. That makes it impossible
to know the initial state, and also the final state.” And he went
on: “We cannot know now whether the final state of our
knowledge shall have a deterministic or a statistical form.
Most physicists today believe that the final form will be of a
probability nature, but I believe the opposite; I believe that it
will be of a deterministic form.”

Not long after the Gibbs lecture, Einstein published in 1935 with Boris Podolsky and
Nathan Rosen their famous paper putting forth their objec-
tions to quantum mechanics, the EPR paradox. Einstein

held to this rearguard position for the rest of his life. (This
paper was submitted before the one on gravity waves and

was accepted by Physical Review.)


David Topper and Dwight Vincent
the theater but being restrained by police; there were also reports of scalpers asking $50 for a ticket. Einstein’s request to limit the audience was prudent in that the Gibbs Lectures were (and still are) directed toward a sophisticated audience knowledgeable in advanced mathematics. Yet he also was cognizant of his celebrity status. Even by limiting the size of the hall he still drew a crowd where few could follow all the steps in an extensive mathematical argument. Our guess is that this crowd was probably the largest attendance of all time for the Gibbs Lectures, which continue today, with the 79th lecture to be delivered at the University of British Columbia (October 4, 2008) by Freeman Dyson of the Institute for Advanced Study.

The late astrophysicist and educator Philip Morrison of the Massachusetts Institute of Technology, an undergraduate physics student at Carnegie Tech at the time, attended the lecture despite having no ticket to enter. Morrison had a friend in the Drama Department who had a key to the theater. They snuck in and climbed onto the scaffolding above the stage, where all they could see was the top of Einstein’s head. Morrison could not see the blackboards. He also reports: “We couldn’t hear very well. But we were there!”¹⁸

The lecture began around 4:30 p.m. and Einstein spoke in English for about one hour (52 minutes, according to one reporter), occasionally querying the audience for an English word. At the end of the lecture someone jumped onto the stage and snatched the chalk used by Einstein. (Today, such an artifact would immediately go up for auction on the Internet.) The headlines in the newspapers the next day were predictable: “Einstein Lectures!/Crowd of 450 Gapes/Then Grows Sleepy” and “Einstein Speaks to 400/But Few Grasp Meaning.”¹⁹ After delivering the Gibbs Lecture, he was surrounded by autograph seekers and others trying to converse with him. But he quickly, and with some difficulty, made his way through the crowd to a waiting car. He was on his way to visit with the Amateur Astronomers Association of Pittsburgh, which had set up an exhibition at the conference. It seems that the club’s president, upon hearing that Einstein was coming to the city, had sent him a letter requesting the visit. He accepted the invitation, and so despite the autograph seekers, Einstein did not want to disappoint the amateur astronomers.¹⁰

By the early 1930s the myths that were to accompany him began to materialize as a personality image was being formed. “A small, sensitive, and slightly naive refugee from Germany ...” one writer called him.¹⁴ An article in the local Jewish newspaper speaks of his theory of relativity as being understood by “only 12 people in the world”—a common myth that persisted through his life. At the same time the image of him as humble and ascetic is revealed when the author speaks of “not only... the clear and intelligent thinking of the scientist but also... the gentle, just humanity of the man as well.” He is “naively modest” as he “leads the life of the simplest sort in the little town of Princeton.”²⁰ Today, as Einstein’s collected papers are published, more previously unknown facts about his personal life reveal how naive this image was.

What did Einstein say to that enthusiastic audience? The diagrams blackboard #1 (Fig. 3) and blackboard #2 (Fig. 4) are spatial representations of the left and right blackboards in Einstein’s talk. We have deduced the equations on these blackboards using the information we could obtain from Fig. 1, the blurry newspaper photograph in Fig. 2, and the sequence of equations given in Einstein’s paper on this
By forcing consistency among these sources we are fairly confident that we have accurately reproduced the blackboard writing that Einstein used at the lecture. We refer the interested reader to Einstein’s write-up\(^5\) for particulars of the calculation and give a brief overview of the talk according to the time order and the blackboard location. We have used the labels [1A], [1B] ... to label the different equation regions of blackboard #1 and the labels [2A], [2B] ... to label the different equation regions of blackboard #2. In Fig. 2 there is some scribbling by Einstein under the equation region [1D]. These scribbles are not in the photograph taken before the lecture and do not seem to correspond to anything in the paper by Einstein. Unfortunately the picture is not clear enough to make out what was written. It may have been part of the banter Einstein had with his audience when he was asked for clarification on several of his terms (like the word “impulse” for instance) during the lecture.\(^21\)

The interpretations of the sections on blackboard #1 (Fig. 3) are as follows.

Section [1A]. Introduction of the standard Minkowski differential line element in terms of the length \(ds\) and the \((x,y,z,t)\) coordinates. Einstein uses units such that \(c^2=1\). The symbol \(u\) inside the square root sign is the magnitude of the velocity of the particle he imagines moving along the path length. The third equation is a statement of a pattern that
Einstein wants to interpret as the relativistic 4-momentum $\eta^\sigma$ in terms of the relativistic energy and the relativistic 3-momentum, respectively. The superscript $\sigma$ runs over all four coordinates, and the subscript $i$ on the velocity $u_i$ varies over the 3D subspace coordinates. The fourth equation is a first-order Taylor-series expansion of the velocity squared that Einstein uses as a Newtonian plausibility check of the component energy/momentum interpretation of the previous equation’s components.

Section [1B]. Statement of the pattern for the relativistic momentum (which Einstein calls “impulse”) and the relativistic kinetic energy. Einstein’s use of the word Impuls, which is German for momentum, is inconsistent with his attempt to speak in English whenever possible. For instance, he uses “Cons. L..” the abbreviation for the English phrase conservation law, in the lower blackboard section [1F]. The abbreviation for kinetic energy appearing in [1B] works in English and German. Einstein is dealing with conserved momentum so the English word “impulse,” which means the change of momentum induced by the sustained application of a force, is not quite appropriate. Nevertheless, the English write-up of the talks uses “impulse” throughout instead of the term “momentum” that would normally be used in English.

Sections [1C] and [1D], Statements of the special Lorentz transformation equations (inverse form) for the time and position, respectively. The prime labels the measurement variables with respect to a frame (called $K'$) moving with speed $v$ relative to another frame (called $K$).

Section [1E], Einstein introduces the idea of a two-particle elastic collision for identical particles of rest mass $m$. The frame $K'$ is assumed to be a three-dimensional zero-momentum frame for this collision. He shows that, given this assumption, summing the velocity patterns appearing in the components of the 4-vector of [1A] over the two particles gives the two displayed equations.

Section [1F], Statement of the conservation of relativistic momentum ($I_\sigma$) and total energy ($E$) before and after the collision. Barred quantities indicate postcollision quantities and the subscript $\nu$ runs over the 3D subspace coordinates. Einstein’s hypothesis is that these conservation laws hold for the relativistic energy and momentum forms introduced in the third equation of [1A].

Section [1G]. Einstein writes a general form for relativistic momentum and total relativistic energy involving general nondirectional velocity functions $F(u)$ and $G(u)$, respectively. These have the property of vanishing at zero velocity. The quantity $E_0$ is the rest energy remaining when the kinetic energy term involving $G(u)$ disappears at zero velocity.

Einstein now makes an argument (see Ref. 5 for the details) involving the zero-momentum frame $K'$ and the result in [1E] to deduce that the interpretations of relativistic momentum and energy expressed in [1B] are correct. Part of Einstein’s motivation in giving the lecture in Pittsburgh was establishing the definition of relativistic momentum and relativistic energy without using electromagnetic theory. The discussion with respect to blackboard #1 accomplished this task.

The equations of blackboard # 2 (Fig. 4) address Einstein’s solution of what he sees as the remaining problem with establishing relativistic energy from purely mechanical considerations. It concerned the interpretation of the rest energy constant term. Because energy is always defined only up to a reference constant, Einstein wanted to show that the rest mass $m$ had some physical significance in the energy expression by deriving an expression that involved changes in the rest mass.

The interpretations of the sections on blackboard #2 (Fig. 4) are as follows.

Section [2A]. Fresh from the argument on blackboard #1 Einstein states the justified form for the relativistic total energy.

Section [2B]. A representation of the zero-momentum frame $K'$.

Section [2C]. By using a nonelastic version of the previous identical particle collision Einstein writes the total energy (before and after the collision) in both the $K$ frame and the $K'$ frame. The symbols + and − are used to label the two colliding particles. The first two equations are transformed into the third (using [1E]) and fourth equations, respectively.

Section [2D]. The last two equations of [2C] are subtracted to give an energy-difference equation.

Section [2E]. The relative velocity $v$ is not zero and therefore the previous difference equation gives that the change in rest energy is equal to the change in the rest mass. Because the energy of any particle is only defined up to a constant, and both $E_0$ and $E_1$ refer to that same arbitrary additive constant, Einstein needed such a difference equation so that the energy constant would cancel out. Because the change in the rest energy is proportional to the change in the rest mass, Einstein concluded that the rest energy must be proportional to the rest mass in general. This conclusion is what the last equation in [2E] states.

What the audience (and undoubtedly, the photographers) wanted to see, of course, is the energy-mass equation in its famous ($c \neq 1$) form

$$E_0 = mc^2.$$  

If combined with the kinetic energy term (the second equation in [1B]) in the pattern given in [1G], the popular modern version of the energy equation would be obtained:

$$E = mc^2,$$

where $m_R = m/\sqrt{1-u^2}$ is the relativistic mass. In this lecture Einstein stayed well clear of defining the relativistic mass parameter, $m_R$, which many relativity experts today see as a nonfundamental construction.

In the end there were several missed opportunities and failed comprehensions that day—surely by the photographers, some from the audience, and perhaps even by Einstein himself. Because he was posed next to the wrong blackboard, no photographer snapped the picture posterity wanted; that is, capturing the famous scientist with his equally famous energy equation. Yet we know that even if he had been placed next to the correct blackboard, few viewers (then, as now) would immediately grasp the significance of $E_0=m$. (How many in the general public know the convention of setting $c=1$?) Moreover, this issue is a subset of a larger one of Einstein’s own making; namely, his decision to overlook the nonmathematicians in the crowd, and instead to pitch his talk consistently at a high mathematical and abstract level. Probably no one expected Einstein to dumb-down his talk; after all, he was delivering the Gibbs Lecture at the request of the AMS. Nevertheless there remains a large middle ground for explaining scientific and mathematical abstractions to an intelligent lay audience, something Einstein himself did in 1916 in his popular book on relativity.
he could have used \( c \) for the speed of light, and thus satisfied the audience with seeing an equation resembling \( E=mc^2 \). He made nary a concession, and thus by opting instead to talk over most of their heads, he missed a pedagogical opportunity. Finally it could be said that he missed a fundamental grasp of the potential of nuclear physics that day when at the news conference he expressed his opinion on the practical impossibility of forging such energy.

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2. Many details of Einstein trip to Pittsburgh are found in the Leon L. Watters papers, manuscript collection #495, from the Jacob Rader Marcus Center of the American Jewish Archives, Cincinnati, Ohio. We are grateful for permission to cite this material (the Watters papers).

3. Figure 1 is found in various sources. It first appeared in the Pittsburgh Sun-Telegraph, December 29, 1934, p. 3. Our version is a copy from a clipping in the Watters papers, box #1, folder #3. Despite considerable effort, we have not been able to find the original photograph. The Sun-Telegraph stopped publication in 1960, and although some original photographs were found in the Library of the University of Pittsburgh, this one of Einstein was not. Our guess is that it and its negative are lost or destroyed.


6. Figure 1 is found in various sources. It first appeared in the Pittsburgh newspapers during Einstein’s visit. There are at least two versions of the same pose, for sometimes his right hand is hanging down and other times his is cocked at a right angle. Different versions are found in different Pittsburgh newspapers, and hence from different photographers. The pictures have been reproduced in several places of which we are aware. Probably the first was on the cover of Sci. Am. 152 (1935). Recent ones are in François Balibar, Einstein: Decoding the Universe (Harry N. Abrams, New York, 2001), pp. 90–91, and John Langone, The Mystery of Time: Humanity’s Quest for Order and Measure (National Geographic Books, Washington, DC, 2000), p. 22.


8. Reference 7, p. 646.

9. It was a joint meeting with the Mathematical Association of America, the American Physical Society, and the American Association for the Advancement of Science.

10. Watters papers, box #1, folder #1.


12. A photograph of Einstein, Robertson, Watters, and others at the news conference may be found on the Caltech Institute Archives website, (www.archives.caltech.edu). Search for photo ID, “LW1-4”.


14. The Literary Digest, January 12, 1935, p. 16 (from the Watters papers, box #1, folder #3).


17. The accused, Bruno Richard Hauptmann, was on trial for abducting and murdering the infant of Charles A. and Anne Morrow Lindbergh in 1932. He was subsequently found guilty and executed in 1936, despite pleading his innocence to the end.


20. The Jewish Criterion, December 28, 1934, p. 5.

21. It was reported that although Einstein mainly spoke in English, there were times when he used German words. At these times members of the audience translated them aloud for Einstein. When he used the term impuls, it was questioned by the audience, and they realized it was the German term for the English term “momentum.” Howard W. Blakeslee, “Mass theory proved anew by Einstein,” Pittsburgh Post-Gazette, December 29, 1934, p. 4.


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