

Cosmology from quantum potential

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It was shown recently that replacing classical geodesics with quantal (Bohmian) trajectories gives rise to a quantum corrected Raychaudhuri equation (QRE). In this article we derive the second order Friedmann equations from the QRE, and show that this also contains a couple of quantum correction terms, the first of which can be interpreted as cosmological constant (and gives a correct estimate of its observed value), while the second as a radiation term in the early universe, which gets rid of the big-bang singularity and predicts an infinite age of our universe.

The generally accepted view of our universe (homogeneous, isotropic, spatially flat, obeying general relativity, and currently consisting of about 72% Dark Energy, likely in the form of a cosmological constant Λ , about 23% Dark Matter, and the rest observable matter), implies its small acceleration, as inferred from Type IA supernova observations, CMBR data and baryon acoustic oscillations [1–4]. However, quite a few things remain to be better understood, e.g.,

- (i) the smallness of Λ , about 10^{-123} in Planck units (*‘the smallness problem’*),
- (ii) the approximate equality of vacuum and matter density in the current epoch (*‘the coincidence problem’*),
- (ii) the apparent extreme fine-tuning required in the early universe, to have a spatially flat universe in the current epoch (*‘the flatness problem’*),
- (iv) the true nature of dark matter, and
- (v) the beginning of our universe, or the so-called big-bang.

In this article, we show that one may be able to get a better understanding of some of the above problems by studying the quantum correction terms in the second order Friedmann equation, derived from the quantum corrected Raychaudhuri equation (QRE), which in turn was obtained by replacing geodesics with quantal (Bohmian) trajectories [5] (This formulation of quantum mechanics gives rise to identical predictions as those of ordinary quantum mechanics). In particular, while one correction term can be interpretable as dark energy, with the right density, and providing a possible explanation of the coincidence problem, the other term can be interpreted as a radiation term in the early universe, preventing the formation of a big-bang type sin-

gularity, and predicting an infinite age of our universe. One naturally assumes a quantum mechanical description of the fluid or condensate filling our universe, described by a wavefunction $\psi = \mathcal{R}e^{iS}$ (assumed normalizable and single valued). Some well-studied examples in curved spacetimes, including in cosmology, include refs.[6–9]. $\mathcal{R}(x^\alpha), S(x^\alpha) =$ real functions), associated with the four velocity field $u_a = (\hbar/m)\partial_a S$, and expansion $\theta = Tr(u_{a;b}) = h^{ab}u_{a;b}$, $h_{ab} = g_{ab} - u_a u_b$ (with vanishing shear and twist, for simplicity. The constant $\epsilon_1 = 1/6$ for conformally invariant scalar fluid, but left arbitrary here.). We will see later in this article that a condensate composed of gravitons with a tiny mass is a natural candidate for this fluid. Then the quantum corrected Raychaudhuri equation follows [10] ¹

$$\frac{d\theta}{d\lambda} = -\frac{1}{3}\theta^2 - R_{cd}u^c u^d + \frac{\hbar^2}{m^2}h^{ab}\left(\frac{\square\mathcal{R}}{\mathcal{R}}\right)_{;a;b} + \frac{\epsilon_1\hbar^2}{m^2}h^{ab}R_{;a;b}, \quad (1)$$

Note that Eq.(1) follows directly the Klein-Gordon or Dirac equation (or the Schrödinger equation for non-relativistic situations), and the quantum corrected geodesic equation that follows from them [10]. The second order Friedmann equation satisfied by the scale factor $a(t)$ can be derived from the above, by replacing $\theta = 3\dot{a}/a$, and $R_{cd}u^c u^d \rightarrow \frac{4\pi G}{3}(\rho + 3p) - \Lambda c^2/3$, [11]

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¹ We use the metric signature $(-, +, +, +)$ here, as opposed to $(+, -, -, -)$ in [10], resulting in opposite sign of the \hbar^2 terms.

² ³ (here the cosmological constant Λ has dimensions of $1/(\text{length})^2$ as usual.)

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda c^2}{3} + \frac{\hbar^2}{3m^2} h^{ab} \left(\frac{\square \mathcal{R}}{\mathcal{R}} \right)_{;a;b} + \frac{\epsilon_1 \hbar^2}{m^2} h^{ab} R_{;a;b}, \quad (2)$$

where the density ρ includes visible and dark matter, and may also include additional densities that arise in massive non-linear theories of gravity [16–18]. The \hbar^2 terms in Eqs.(1) and (2) represent quantum corrections (the first of these is also known as quantum potential), which vanish in the $\hbar \rightarrow 0$ limit, giving back the classical Raychaudhuri and the Friedmann equations. Note that these additional terms are not ad-hoc or hypothetical, but rather an unavoidable consequence of a quantum description of the contents of our universe. Also, since it is well known that Bohmian trajectories do not cross [19, 20], it follows that even when θ (or \dot{a}) $\rightarrow -\infty$, the actual trajectories (as opposed to geodesics) do not converge, and there is no counterpart of geodesic incompleteness, or the classical singularity theorems, and singularities such as big bang or big crunch are in fact avoided. This view is also supported by the quantum corrected geodesic deviation equation derived in [10], which suggested that trajectories can never actually access infinite curvatures ⁴. We will return to this issue later, and consider the first of these terms, which naturally appears as a cosmological constant

$$\Lambda_Q = \frac{\hbar^2}{m^2 c^2} h^{ab} \left(\frac{\square \mathcal{R}}{\mathcal{R}} \right)_{;a;b}. \quad (3)$$

Λ_Q depends on the amplitude \mathcal{R} of the wavefunction ψ , which we take to be the macroscopic ground state of a condensate (more on the details of condensate in [21]). Its exact form is not important to our argument however, except that it is non-zero and spread out over the range of the observable universe. This follows from the requirement of causality; even if matter exists beyond the horizon, it will have no effect on what is inside the horizon, including the wavefunction. To estimate Λ_Q , one may assume a Gaussian form $\psi \sim \exp(-r^2/L_0^2)$, or for one which results when an interaction of strength g is included in a

scalar field theory, such that $\psi = \psi_0 \tanh(r/L_0\sqrt{2})$ ($g > 0$) and $\psi = \sqrt{2} \psi_0 \text{sech}(r/L_0)$ ($g < 0$) [23], it can be easily shown that $(\square \mathcal{R}/\mathcal{R})_{;a;b} \approx 1/L_0^4$, where L_0 is the characteristic length scale in the problem, which is of the order of the Compton wavelength $L_0 = h/mc$ [24], over which the wavefunction is non-vanishing. This gives

$$\Lambda_Q = \frac{1}{L_0^2} = \left(\frac{mc}{\hbar} \right)^2, \quad (4)$$

which has the correct sign as the observed cosmological constant. Next to estimate its magnitude, we identify L_0 with the current linear dimension of our observable universe, since anything outside it would not influence an accessible wavefunction. With this, m can be regarded as the small mass of gravitons (or axions), with gravity (or Coulomb field) following a Yukawa type of force law $F = -\frac{Gm_1 m_2}{r^2} \exp(-r/L_0)$. Since gravity has not been tested beyond this length scale, this interpretation is natural, and may in fact be unavoidable [21]. If one invokes periodic boundary conditions, this is also the mass of the lowest Kaluza-Klein modes. Substituting $L_0 = 1.4 \times 10^{26} m$, one obtains $m \approx 10^{-68} kg$ or $10^{-32} eV$, quite consistent with the estimated bounds on graviton masses from various experiments [25], and also from theoretical considerations [26–29]. In other words, we interpret the quantum condensate as made up of these gravitons, and described by a macroscopic wavefunction. Finally, plugging in the above value of L_0 in Eq.(4), we get

$$\begin{aligned} \Lambda_Q &= 10^{-52} (\text{metre})^{-2} & (5) \\ &= 10^{-123} (\text{in Planck units}), & (6) \end{aligned}$$

which indeed matches the observed value. Also, since the size of the observable universe is about c/H_0 , where H_0 is the current value of the Hubble parameter [31], one sees why the above value of Λ_Q numerically equals H_0^2/c^2 (which is $8\pi G/3c^4 \times \rho_{crit}$, the critical density), offering a viable explanation of the coincidence problem. Note that the above also implies that this equality will hold at all times during the evolution of the universe. While the above relationship may have been known, here we provide a natural explanation for it, in terms of quantum corrections to the Raychaudhuri equations via the wavefunction of the cosmic fluid. This also brings out the essential quantum origin of the small cosmological constant.

One may also be tempted to interpret the quantum potential term as a perfect fluid with $w_Q \equiv p_Q/\rho_Q = -1/3$ (such that $\rho_Q \propto a^{-3(1+w_Q)} \propto a^{-2}$). However, a careful analysis using chi-squared techniques, of the luminosity distances versus red-shifts of 580 union 2.1 Supernovae data points, baryon acoustic oscillations, Hubble and CM shift parameters suggest that the corresponding density does not exceed 5% of the current density of the universe (at 95% confidence level), and also does not play any significant role in early or late epochs [30]. We therefore do not consider that possibility.

² This procedure, as well the rest of the paper assumes large scale homogeneity and isotropy. Even if there are small (perturbative) deviations from homogeneity, these can be absorbed in an effective density ρ . Further these do not affect the dark energy content and accelerated expansion of the universe [12, 13].

³ Note that in [14] too, the authors studied dark energy from the Bohmian mechanics perspective, but originating in a scalar field with non-standard action. Also, recently in [15], the authors used Bohmian mechanics in the context of Wheeler-DeWitt equation, to explain inflation.

⁴ A similar conclusion was also arrived at by the frequency dependence of light paths ('gravity's rainbow') picture in [22].

Next, we consider the second correction term in Eq.(2), which can be written in terms of $H = \dot{a}/a$, and for one species of fluid, with $p/\rho = w$ as ⁵

$$\begin{aligned} \dot{H} &= -\frac{3}{2}(1+w)H^2 + \frac{\epsilon_1 \hbar^2}{m^2} h^{ab} R_{;a;b} \\ &= -\frac{3}{2}(1+w)H^2 \\ &\quad - \frac{\epsilon_1 \hbar^2}{m^2} 6H^4(1+w) \left[6(1+w)^2 - \frac{81}{2}(1+w) + 18 \right] \end{aligned} \quad (7)$$

where in the last step we have plugged in the FRW metric. When $\hbar = 0$, integration yields $H \rightarrow \infty$ in a finite time, signifying a big-bang type of singularity. It is interesting to note that H^4 proportional terms were also obtained from (i) the trace anomaly of a conformal field theory dual to a five-dimensional Schwarzschild-AdS geometry, and which is known as holographic/conformal-anomaly Friedmann equation [33, 34], (ii) correction to Raychaudhuri equation in cosmology derived in brane world scenarios [35], and (iii) derived in spacetime thermodynamics and the generalized uncertainty principle of quantum gravity [34]. It would be interesting to investigate underlying connections between the above if any.

Next, to examine the presence or absence of past singularities, we write Eq.(7) as

$$\dot{H} = F(H) , \quad (8)$$

and from which the age of the universe as

$$T = \int_0^T dt = \int_{H_0}^{H_P} \frac{dH}{F(H)} , \quad (9)$$

where H_0 signifies the current epoch. For the ordinary FRW universe with no quantum corrections of the above type (i.e. $\epsilon_1 = 0$), $F(H) = -(3/2)(1+w^2)H^2$, the density and $H \rightarrow \infty \equiv H_P$ in the past (big-bang singularity), where $|\dot{H}| \rightarrow \infty$ as well, and we get

$$T = \frac{2}{3(1+w)^2 H_1} , \quad (10)$$

which once again, is finite. Now if corrections to the classical FRW model changes the nature of the function $F(H)$ (e.g. the degree of the polynomial), such that now neither H nor \dot{H} diverges, then if H_P signifies the nearest fixed point in the past, such that $F(H_P) = 0$, we approximate $F(H) = F^{(n)}(H_P)(H - H_P)^n$ near the fixed point, the region which contributes most to the integral, and obtain [36]

$$T = \frac{1}{F^{(n)}(H_P)} \int_{H_1}^{H_P} \frac{dH}{(H - H_P)^n} \rightarrow \infty , \quad (11)$$

⁵ The following discussions, and in particular the conclusion following from Eq.(11), remain valid even if the first (cosmological constant) term is retained.

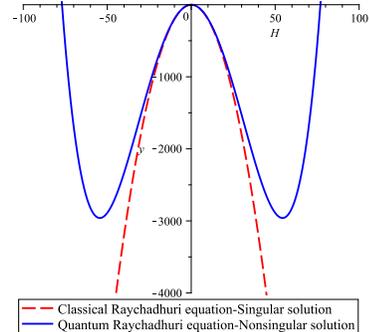


FIG. 1: \dot{H} versus H

signifying a universe without a beginning. This is precisely what is expected from the no-focusing of geodesics and the quantum Raychaudhuri equation. For example, for Eq.(2), it can be easily shown that the sign of the quantum correction (H^4) term is positive (i.e. opposite to the classical, H^2 term), for $-0.52 \leq w \leq 5.27$, which covers most of the physically interesting range, including $w = w_R = 1/3$ (radiation), which is most relevant for the very early universe, and also non-relativistic matter ($w = w_{NR} = 0$). The situation is depicted in Figure (1), where it can be seen that in the above range of w , H_P is indeed finite, and therefore T is infinite from Eq.(11). Thus, the second quantum correction in the Friedmann equation gets rid of the big-bang singularity.

In summary, we have shown here that as for the QRE, the second order Friedmann equation derived from the QRE also contains two quantum correction terms. These terms are generic and unavoidable and follow naturally in a quantum mechanical description of our universe. Of these, the first can be interpreted as cosmological constant or dark energy of the correct (observed) magnitude and a small mass of the graviton (or axion). The second quantum correction term pushes back the time singularity indefinitely, and predicts an everlasting universe. While inhomogeneous or anisotropic perturbations are not expected to significantly affect these results, it would be useful to redo the current study with such small perturbations to rigorously confirm that this is indeed the case. Also, as noted in the introduction, we assume it to follow general relativity, whereas the Einstein equations may themselves undergo quantum corrections, especially at early epochs, further affecting predictions. Given the robust set of starting assumptions, we expect our main results to continue to hold even if and when a fully satisfactory theory of quantum gravity is formulated. For the cosmological constant problem at late times on the other hand, quantum gravity effects are practically absent and can be safely ignored. We hope to report on these and related issues elsewhere.

Acknowledgment

We thank S. Braunstein, M. W. Hossain, S. Kar, M. Sami, T. Sarkar and S. Shankaranarayanan for useful comments. We thank the anonymous referee for useful comments. This work is supported by the Natural Sci-

ences and Engineering Research Council of Canada, and the Perimeter Institute for Theoretical Physics, through their affiliate program. The research of A. Farag Ali is supported by Benha University (www.bu.edu.eg) and CFP in Zewail City, Egypt.

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