Kepler's 1st Law

"Now between the circle and the ellipse there is no other intermediary except a different ellipse. Therefore the path of the planet is an Ellipse ..." - Johannes Kepler (1571 - 1630)

[Source: **"**The New Astronomy**"**: Astronomia nova (Heidelberg, 1609) Chapter 58, 284 - 85, KGW 3 366, from School of Mathematics and Statistics, University of St Andrews, Scotland]

Johannes Kepler (1571 – 1630)
German mathematician, astronomer and astrologer
portrait circa 1610 - artist unknown

§ Kepler's 1st Law (Planetary Law of Ellipses: Sun - centered model):

All planetary orbits are ellipses with the Sun at one of the two foci.

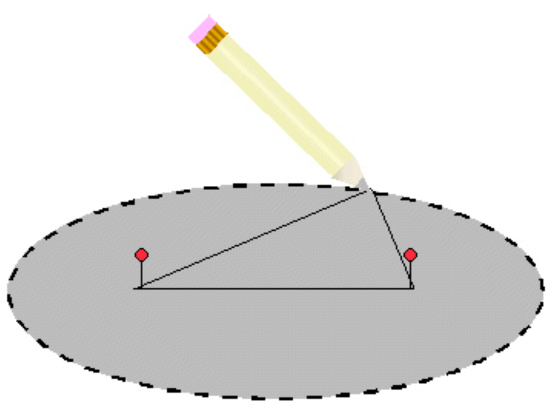
An *ellipse* is defined as the locus of points, the sum of whose distance from two fixed points (the foci) is constant. That is, an ellipse is a special curve where the sum of the distances from every point on the curve to two other points is a fixed constant.

The ellipse equation is therefore

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

An ellipse is drawn by using two tacks into a piece of cardboard with a taut string and by moving a pencil held just inside the string.

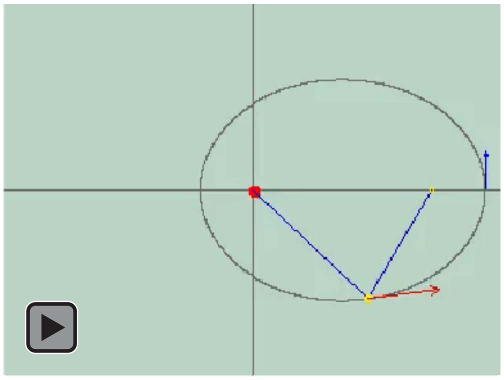
Using this picture you can draw an ellipse as follows:

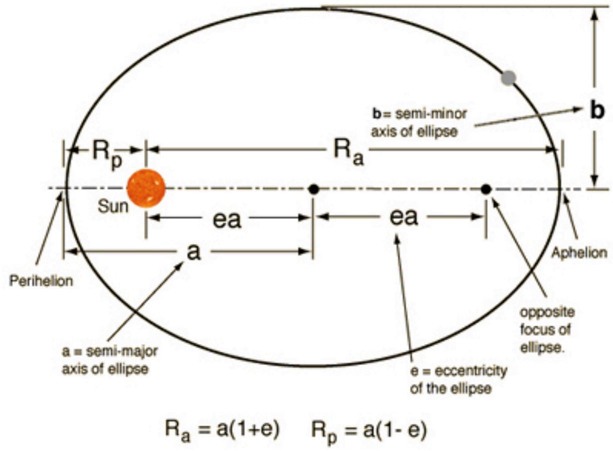


The closer together which these points are, the more closely that the ellipse resembles the shape of a circle. In fact, a circle is the special case of an ellipse in which the two foci are at the same location.
Where the two tacks (foci) are come closer, the ellipse will approach a circle. In fact, every circle is a special case ellipse where the two foci are identical.

Hence,

e = eccentricity of a circle = ea/a = c/a = 0. See below for this definition.
0 < eccentricity of an ellipse < 1
eccentricity of a straight line = ∞, infinity [∞ = lemniscate, latin for ribbon]





- Sun = red circle, one of two foci; stationary yellow circle is an imaginary 2nd foci
- Planet = moving yellow circle
- Blue arrow = initial condition
- Red arrow = moving planet and is proportional to planet's velocity
- The Perigee: Closer to the sun, the faster the planet passes in its transit orbit
- The Apogee: Further from the sun, the slower the planet passes in its transit orbit

[note: the last two observations hold because of Kepler's 2nd Law of Equal Areas where a planet sweeps out equal areas during equal intervals of time.]

Sun at one of the two foci
Major Axis = Rp + Ra = 2a
Minor Axis = 2b
a = Semi-major axis of ellipse
Rp = perihelion radius
Ra = aphelion radius
Rav = a = 1/2(Ra + Rp) = average orbital radius
c =ea = 1/2(Ra - Rp) = interfocal radius
e = eccentricity of ellipse = ea/a = c/a.

source: http://dev.physicslab.org/Document.aspx?doctype=3&filename=OscillatoryMotion_KeplersLaws.xml

§ Equations for Planetary Orbital Eccentricity:

$$e = \sqrt{1 + \frac{2Eh^2}{G^2M^2m}} = \frac{p}{r_o} - 1 = \frac{r_o v_o^2}{GM} - 1, \text{ eccentricity}$$

$h \equiv r^2\dot{\theta} = r_o v_o$, $\underline{\text{constant angular momentum}}$ per unit mass at the initial boundary condition of "perihelion time" $t_o = 0$ where also instantaneous minimum value r at perihelion produces $\dot{r} = 0$

$r_o = \min(r)$ at "perihelion time" $t_o = 0$

$$p = a(1 - e^2) = \frac{h^2}{GM}$$

$E \equiv m\varphi(\mathbf{r}) = m\varphi(x, y, z) = -G\dfrac{mM}{r}$, $\underline{\text{gravitational potential energy of a 2 - body } (m, M)}$ $\underline{\text{system}}$ separated by a $\mathbf{r} \equiv \vec{r}$ radial distance, where $r = |\mathbf{r}| = |\vec{r}|$ and $E < 0$, $\underline{\text{negative energy}}$ for ellipses

§ See the pdf on the Proof for Kepler's 1st Law.

§ References:

Kepler's 1st Law (Planetary Law of Ellipses: Sun - centered model) "Astronomia Nova - 1609", by Johannes Kepler (1571 - 1630)

Kepler's 1st Law (Planetary Law of Ellipses: Sun - centered model) "Harmonices Mundi - 1619", by Johannes Kepler (1571 - 1630)