Maxwell's Equations

§ Maxwell's Equations by James Clerk Maxwell:

From his original 1861 "On Physical Lines of Force"

§ Defining terms:

$$\vec{E} = (t, x, y, z), \text{ electric field strength vector at point } (x, y, z) \text{ at time } t$$
$$\vec{H} = (t, x, y, z), \text{ magnetic field strength vector at point } (x, y, z) \text{ at time } t$$
$$\vec{D} = \epsilon \vec{E}, \text{ electric displacement vector}$$
$$\vec{B} = \mu \vec{H}, \text{ magnetic induction vector}$$
$$\epsilon = \begin{cases} 1 & : \text{ empty space} \\ > 1 & : \text{ non - empty space} \end{cases}, \text{ dielectric constant}$$
$$\mu = \text{magnetic permeability (penetrability)}$$
$$\rho = \text{electric charge density}$$
$$\vec{j} = \text{electric current vector}$$
$$\text{div}, \ \nabla^{*} = \vec{i}_1 \frac{\partial}{\partial x_1} + \vec{i}_2 \frac{\partial}{\partial x_2} + \vec{i}_3 \frac{\partial}{\partial x_3} + \vec{i}_4 \frac{\partial}{\partial x_4}, \text{ vector operator - read: "divergence" or "del" respectively}$$
$$\text{curl} = \nabla \times, \text{ circulation density of a vector field}$$

*note: this vector operator gives the variation (gradient) of the field in which it operates at every point of a well - defined gradient field

§ Maxwell's equations:

$$\text{div } \epsilon \vec{E} = 4\pi\rho \quad \text{or} \quad \nabla \cdot \vec{D} = 4\pi\rho, \text{ Poisson's Equation} \tag{1}$$
$$\text{div } \mu \vec{H} = 0 \quad \text{or} \quad \nabla \cdot \vec{B} = 0, \text{ no name} \tag{2}$$
$$c \text{ curl } \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad \text{or} \quad \nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}, \text{ Faraday's Law} \tag{3}$$
$$c \text{ curl } \vec{H} = \epsilon \frac{\partial \vec{E}}{\partial t} + 4\pi\vec{j} \quad \text{or} \quad \nabla \times \vec{H} = \frac{1}{c}\frac{\partial \vec{D}}{\partial t} + \frac{4\pi}{c}\vec{j} = \frac{4\pi}{c}\left(\vec{j} + \frac{1}{4\pi}\frac{\partial \vec{D}}{\partial t}\right), \text{ Maxwell - Ampere Law} \tag{4}$$

where the displacement of current, $\frac{1}{4\pi}\frac{\partial \vec{D}}{\partial t}$, was confirmed by Hertz's experiments

§ Lorentz Force Law:

$$\vec{F} = q\left(\vec{E} + \frac{1}{c}\vec{v} \times \vec{B}\right), \text{ Lorentz Force Law}$$

where

$\vec{F}$ = force exerted on a charged particle in an electromagnetic field

q = electric charge

c = speed of light *in vacuo*

$\vec{v}$ = instantaneous velocity of charged particle in an electromagnetic field

$\times$ = vector cross product - i.e., not your ordinary 'times' multiplication

§ Unifying all of electrodynamics:

What makes for the unity of all of electrodynamics is Maxwell's equations plus Lorentz Force Law.

§ Interpreting Maxwell's equations:

equation (1): In the presence of an electric charge, the resulting electric field will be such that the electric charge will be exactly compensated by the electric displacement;

equation (2): An equal amount of magnetic displacement passes outward thru every closed surface as it passes inward into the same closed surface - i.e., that the magnetic field lines must be closed loops since there are no such things in nature as free magnetic charges;

equation (3): Every magnetic displacement current has a reverse sense electric field;

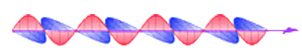
equation (4): Every electric current produces its own magnetic field.

Maxwell wrote in his **"**A Dynamical Theory of the Electromagnetic Field**"** (1864), pg. 499, part (97), that

> **"***The agreement of the results seems to show that light and magnetism are affections of the same substance, and that light is an electromagnetic disturbance propagated through the field according to electromagnetic laws.***"**

§ Maxwell's Theory of the Electromagnetic Field:

According to Maxwell's equations, electromagnetic radiation (EM) is a self - propagating transverse wave with electric and magnetic components where the oscillating electric and magnetic field components induce their respective opposites and vice-versa. These electric and magnetic components oscillate at right angles to each other and propagate perpendicularly to the direction in which they indefinitely travel unless absorbed by intervening matter. That is, each kind of field - electric and magnetic - generates the other in order to propagate the entire composite structure moving forward through empty space at the finite speed of light, c. In order of increasing frequency, the types of electromagnetic waves include radio and television waves; microwaves; infrared radiation; visible light; ultraviolet light; x - rays; gamma rays; and finally, cosmic rays.



§ Maxwell: "On Faraday's Lines of Force", Transactions of the Cambridge Philosophical Society, Vol. X, Part I - read on Dec. 10, 1855 and Feb. 11, 1856