THE DERIVATION OF THE RADIAL VELOCITY EQUATION

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The visual observer, in endeavoring to grade all the stars in classes, may take too large a difference of brightness to represent the distriction of a quarter of a magnitude, a very small quantity to be estimated, as he goes down in the scale beyond the types of stars from which he must form his magnitude scale. The photometric measures may have developed a tendency in the contrary direction; taking too small a difference of scale reading to represent the difference of a tenth, or of a quarter of a magnitude. The photometer has the great advantage that large differences of brightness can be measured, and used as checks, assuming that the instrument gives consistent and authentic values for the steps down from one type of star to another, at all parts of the scale of brightness.

A great amount of detailed comparison of the photometric and visual D. M. magnitudes has been given in the latest publication of the Harvard College Observatory, Parts 6 and 7 of Volume 72, which were received here after the figures of this note had been prepared.[1]

July 12, 1913.

THE DERIVATION OF THE RADIAL VELOCITY EQUATION.

By G. F. Paddock.

The radial velocity equation representing the component of velocity of binary stars in the direction of the Sun was first derived by Lehmann-Filhes.[2] His method, which follows in brief, was the differentiation of the co-ordinate z, whose origin is at the center of motion and whose direction is along the line of sight. Let—

a = semi-major axis;
e = eccentricity;
i = inclination of the orbit plane with respect to the plane tangent to the celestial sphere;
ω = longitude of periastron from the receding or ascending node;

[1] In Campbell's "Stellar Motions," recently issued from the Yale University Press, he refers to the number of stars observable in our greatest telescopes as being about one hundred million. (Page 118.)

[2] *Astronomische Nachrichten*, **136**, 17.

$u = v + \omega =$ longitude in the orbit measured from the receding node in the direction of motion;
$v =$ true anomaly;
$p =$ semi-parameter $= a\,(1 - e^2)$;
$\mu =$ mean daily motion;
$r =$ relative radius vector;
$t =$ time;
$f =$ constant of attraction in the system;
$k =$ Gaussian constant for the solar system;
$m =$ mass of either component;
$M = m_1 + m_2 =$ total mass;
$K =$ semi-amplitude of velocity oscillation;
$V =$ radial velocity in kilometers per second relative to the Sun,—

Then

$$Z = r \sin i \sin u,$$

$$\frac{dz}{dt} = \sin i \sin u \frac{dr}{dt} + r \sin i \cos u \frac{du}{dt}.$$

Taking from the theory of orbital motion in a plane the expressions

$$\frac{dr}{dt} = \frac{f}{\sqrt{p}} e \sin (u - \omega),$$

$$r\frac{du}{dt} = \frac{f}{\sqrt{p}} (1 + e \cos [u - \omega]),$$

we derive

$$\frac{dz}{dt} = \frac{f}{\sqrt{p}} \sin i\, (\cos u + e \cos \omega),$$

or

$$V = K\,(\cos u + e \cos \omega).$$

It is obvious that this equation should be derivable from the fundamental equations of motion and it may be of interest to show how it may thus be derived, although a simple process.

The fundamental equations for the z components of motion of a binary system are

$$\frac{d^2 z_1}{dt^2} = - k^2 m_2 \frac{z_1 - z_2}{r^3},$$

$$\frac{d^2 z_2}{dt^2} = - k^2 m_1 \frac{z_2 - z_1}{r^3}.$$

Since radii vectores are inversely proportional to the masses, we have

$$r_1 + r_2 = r = \frac{m_1 + m_2}{m_2} r_1 = \frac{m_1 + m_2}{m_1} r_2,$$

$$z_1 - z_2 = z = \frac{m_1 + m_2}{m_2} z_1 = -\frac{m_1 + m_2}{m_1} z_2.$$

Hence we have

$$\frac{d^2 z_1}{d t^2} = -k^2 \frac{m_2^3}{(m_1 + m_2)^2} \frac{z_1}{r_1^3},$$

$$\frac{d^2 z_2}{d t^2} = -k^2 \frac{m_1^3}{(m_1 + m_2)^2} \frac{z_2}{r_2^3},$$

$$\frac{d^2 z}{d t^2} = -k^2 (m_1 + m_2) \frac{z}{r^3}.$$

The last equation applies to relative motion. The coefficients on the right-hand side represent the constant of attraction for each case, as follows:—

$$f_1 = k \frac{m_2^{\frac{3}{2}}}{m_1 + m_2}, \quad f_2 = k \frac{m_1^{\frac{3}{2}}}{m_1 + m_2}, \quad f = k \sqrt{m_1 + m_2}.$$

The equations are therefore of the same form and give exactly similar integrals and orbits. The integration need be performed for only one,— for example, the last,—and may be effected with the help of the integrals of relative motion in the orbit plane, namely,—

the orbit equation,

$$r = \frac{p}{1 + e \cos (u - \omega)},$$

the areal velocity,

$$r^2 \frac{d u}{d t} = k \sqrt{M} \sqrt{p},$$

the orbital radial velocity,

$$\frac{d r}{d t} = \frac{r^2 e \sin (u - \omega)}{p} \frac{d u}{d t} = \frac{k \sqrt{M}}{\sqrt{p}} e \sin (u - \omega).$$

Then since

$$z = r \sin i \sin u,$$

the last of the equations of motion becomes

$$\frac{d^2 z}{d t^2} = - \frac{k \sqrt{M}}{\sqrt{p}} \sin i \sin u \frac{d u}{d t}.$$

Integration gives

$$\frac{d z}{d t} = \frac{k \sqrt{M}}{\sqrt{p}} \sin i \cos u + c.$$

To determine the constant of integration c, let $u = \frac{1}{2}\pi$. Then

$$c = \left.\frac{d z}{d t}\right]_{u=\frac{1}{2}\pi} = \sin i \sin u \left.\frac{d r}{d t}\right]_{u=\frac{1}{2}\pi} = \frac{k \sqrt{M}}{\sqrt{p}} \sin i . e \cos u.$$

Then putting

$$V = \frac{d z}{d t} \text{ and } K = \frac{f \sin i}{\sqrt{p}},$$

we have

$$V = K \ (\cos u + e \cos \omega).$$

The V and K may now refer to either relative motion or to motion of either component with respect to the center of mass.

Since

$$\mu a^{\frac{3}{2}} = f,$$

we have, if k be given for the unit of time one second,

$$K = k_s \sqrt{M} \frac{\sin i}{\sqrt{p}} = \frac{\mu}{86400} \frac{a \sin i}{\sqrt{1 - e^2}}$$

$$K_1 = \frac{k_s m^{\frac{3}{2}}}{m_1 + m_2} \frac{\sin i}{\sqrt{p_1}} = \frac{\mu}{86400} \frac{a_1 \sin i}{\sqrt{1 - e^2}}$$

$$K_2 = \frac{k_s m^{\frac{3}{2}}_1}{m_1 + m_2} \frac{\sin i}{\sqrt{p_2}} = \frac{\mu}{86400} \frac{a_2 \sin i}{\sqrt{1 - e^2}}$$

Then since $\omega_2 = \omega_1 + \pi$,

and if V_1 and V_2 are the velocities relative to the Sun and γ the velocity of the center of mass relative to the Sun, we have for the velocity equations of each component of a binary system,

$$V_1 = \gamma + K_1 \ (\cos u + e \cos \omega),$$
$$V_2 = \gamma - K_2 \ (\cos u + e \cos \omega).$$

May, 1913.